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LECTURES



ON THE

WAVE-THEORY OF LIGHT.

BY

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CONTENTS.

PART I. UNPOLARIZED LIGHT.

LECTURE I.

	Page.
Propagation of Light. Reflexion and Refraction.	5

LECTURE II.

Interference of Light.	25
--------------------------------	----

LECTURE III.

Diffraction of Light. Newton's Theory of Inflexion.	41
---	----

LECTURE IV.

Diffraction of Light—Theory of Young and Fresnel.	51
---	----

LECTURE V.

Colours of thin Plates. Newton's Hypothesis of the Fits of easy Reflexion and Transmission.	68
--	----

LECTURE VI.

Colours of thin Plates—Theory of Young. Colours of thick Plates.	78
--	----

PART II.

POLARIZED LIGHT.

LECTURE I.

	Page.
Double Refraction—Phenomena.	1

LECTURE II.

Polarization of Light—Phenomena.	12
--	----

LECTURE III.

Polarization of Light—Principle of transversal Vibrations. Theory of Reflexion and Refraction of polarized Light.	24
--	----

LECTURE IV.

Double Refraction—Fresnel's Theory. Conical Refraction.	42
---	----

LECTURE V.

Interference of Polarized Light. Colours of Crystalline Plates. Rings surrounding the optic Axes of Crystals.	59
--	----

LECTURE VI.

Interference of polarized Light—Rotatory Polarization. Double- refracting Structure produced by mechanical Compression, or unequal Temperature.	73
---	----

LECTURE I.

PROPAGATION OF LIGHT.—REFLEXION AND REFRACTION.

GENTLEMEN,—The phenomena of nature may be studied under two different aspects, which it is often convenient to keep separate. The comparison of individual facts may lead to the discovery of a general law, in which we may rest as an *ultimate fact* in science, and content ourselves with tracing its consequences in an endless variety of new cases, until we have created a branch of science of which it is the fountain and head. Or, looking higher, we may endeavour to refer this law itself to some simpler property of matter, and, combining a few principles of the highest generality, we may form a *theory* from whose bosom we may evolve these and other subordinate laws. Each of these processes has been pursued in astronomy. You are aware that in one mode of studying that interesting science, the laws of Kepler are assumed as fundamental principles, and from them, when unfolded, all the more obvious appearances of the planetary system are deduced. In *physical astronomy*, on the other hand, the laws of Kepler themselves are derived as consequences of a primordial law of matter, and the development of this higher principle has brought to light a multitude of other laws of the universe, which mere observation could have never reached.

A similar division may be made in the study of the phenomena of light. In the preceding part of our course we adopted the former of these methods; we took the fundamental laws of reflexion and refraction as principles borrowed from experience, and raised by the process of induction; and we saw that they not only explained a multitude of natural phenomena,—but that, guided by the results to which we were conducted in their mathematical development, man has been enabled to contrive new combinations serving important ends, and to invent instruments which have wonderfully extended the sphere of his knowledge and power. The same course might be pursued also with respect to other phenomena of light, which the discoveries of later years have brought forth; and to some such a course may possess its attractions. But I prefer at once to endeavour to lead you to that more elevated station in science, whence all these varied phenomena and their laws may be embraced in a single view; and to attempt to develop a *physical theory* of light, which has not only accounted for the greater part of the known facts, but even in many cases anticipated observation.

It is well known that light travels from one point of space to another *in time*, and that the velocity of this progressive movement amounts nearly to that of 200,000 miles in a second. Now, there are two distinct and intelligible ways of conceiving such a propagated movement. Either it is the *same individual body* which is found in different times in distant parts of space;—or there are a *multitude* of moving bodies, occupying the entire interval, each of which *vibrates* continually within certain limits, while the vibratory motion itself is communicated in succession from one to another, and so advances uniformly. These two modes of propagated movement may be distinguished by the names of the motion of *translation* and the motion of *vibration*. The former is more familiar to our thoughts, and is that which we observe,

when with the eye we follow the path of a projectile in the air; or about which we reason, when we determine the course of a planet in its orbit. Motions of the latter kind, too, are every where taking place around us. When the surface of stagnant water is agitated by any external cause, the particles of the fluid next the origin of the disturbance are set vibrating up and down, and this vibratory motion is communicated to the adjacent particles, and from them onwards, to the boundaries of the fluid surface. All the particles which are elevated at the same instant constitute what is called a *wave*; and that this wave does not consist of the *same* particles in two successive instants may be seen in the movements of any floating body, which will be observed to rise and fall as it is reached and passed by the wave, but not to advance, as it must necessarily do if the particles of the fluid on which it rested had a progressive motion. The phenomena of sound afford another well known instance of the motion of vibration. The vibratory motion is communicated from the sounding body to the ear, through all the intervening particles of the air, though each of the aerial particles moves back and forwards through a very narrow space.

Each of these modes of propagated motion has been applied to explain the phenomena of light; and hence the two rival theories—the *theory of emission* and the *wave-theory*. In the former the luminous body is supposed to send forth, or *emit*, particles of extreme minuteness, in all directions. In the latter, the same body is supposed merely to *excite the vibrations* of an *elastic ether*, just as a sounding body excites the vibrations of the air. This ethereal medium is supposed to pervade all space, and to be of such extreme tenuity as to afford no appreciable resistance to the motions of the planets.

Such are the two systems, some traces of which may be found even in the recorded opinions of the ancients. It is only within a period comparatively recent, however,

that either of them has been stated with distinctness, or supported by any show of reasoning. The great advocate,—we may almost say—the author of the theory of emission, was Newton, to whom (it has been already seen) we owe our knowledge of some of the most important properties of light. The wave-theory was unfolded, and applied to phenomena a short time before, by Grimaldi and Huygens. In the course of the succeeding lectures I propose to review in succession the principal phenomena of ordinary, or unpolarized light, and to examine each of these theories by the only test to which a physical theory can be subjected,—namely, the accordance of its consequences with established facts.

The fundamental assumption of the theory of emission—the hypothesis that the rays of light are *bodies* moving with an immense velocity—would appear to be easily submitted to the test of experiment. If the weight of a molecule of light amounted to but *one grain*, its momentum would equal that of a cannon ball, 150 pounds in weight, and moving with the velocity of 1000 feet in a second. The weight of a single molecule may be assumed to be many millions of times less than what has been here supposed; but, on the other hand, many millions of such molecules may be made to act together, by concentrating them in the foci of lenses or mirrors, and the effects of their impulse might be expected in this manner to be rendered evident. This apparently easy test of the materiality of light was appealed to by many experimental philosophers of the last century, and with various results. The effects observed by Homborg, Michell, and others, have been traced, however, with much probability to extraneous causes, (such as aerial currents produced by unequal temperature;) and it is now universally conceded that no sensible effect of the *impulse* of light has been ever perceived. The experiments of Mr. Bennet seem to be decisive on this point. In these experiments a slender straw was suspended horizontally by means of a single fibre of the

spider's thread. To one end of this delicately suspended lever was attached a small piece of white paper, and the whole was inclosed in a glass vessel, from which the air was withdrawn by the air-pump. The sun's rays were then concentrated by means of a large lens, and suffered to fall upon the paper, but without any perceptible effect.

But these experiments, though they have failed in ad-
ducing evidence in favour of the material hypothesis, cannot,
on the other hand, be said to be conclusive against it.
However small the limit which may be thus assigned to the
momentum of a single particle of light, it is easy to attribute
to it a *mass* of corresponding minuteness; and in whatever
point of view we contemplate the phenomena of light, we are
forced to deal with magnitudes which almost bewilder the
imagination. It is only by the effects of light *upon light* (it
appears to me) that the question of its materiality can be
fairly tried by direct experiment; and I have long thought
that the simple fact of the crossing of the rays at the focus of
a lens *without disturbance* is decisive on the question. There
are certain phenomena which (it will be presently shown)
compel the advocates of the theory of emission to suppose,
that the molecules of light are not shot off from the lumi-
nous origin at random, but at *regular intervals*; so that
they must form concentric spherical surfaces. If then a lens
be placed so as to receive the rays perpendicularly, it is ob-
vious that *all* the molecules of one of these surfaces, which
fall upon any one annulus of the lens, *must* meet at its focus.
It is needless to dwell upon the consequences of such an im-
pact. There is nothing in the phenomena of refraction which
can justify the assumption that it has taken place.

But the actual velocity of light is not the only difficulty
which the theory of emission has to encounter at the very
outset. The discoveries of astronomy have further revealed
that this velocity is *one and the same*, whether the light is
directly emitted from the sun or a fixed star, or reflected

from a planet or its satellite; that the velocity of light is, in short, independent of the luminous source, as well as of the subsequent modifications which it undergoes in the celestial spaces. It is not easy to account for these facts in the theory of emission. The emissive force required to produce the known velocity is calculated to be more than a million of million times greater than the force of gravity at the earth's surface; and is it to be supposed that this prodigious force is *the same* for all the various and independent bodies of the starry universe, and that it acts *equally* on all the particles of light, so as to generate in all the same velocity? Yet even this assumption, unnatural as it is, will not avail. Laplace has shown, that if the diameter of a fixed star were 250 times as great as that of our sun, its density being the same, its attraction would be sufficient to destroy the whole momentum of the emitted molecules, and the star would be invisible at great distances. With a smaller mass there will be of course a proportionate retardation, so that the final velocities will be different, whatever be the initial ones. The suggestion of M. Arago seems to offer the only way of escaping the force of this objection. It may be supposed that the molecules of light are originally projected with very different velocities, but that among these velocities there is but one which is adapted to our organs of vision, and which produces the sensation of light. This supposition seems to be supported by the discoveries of Herschel, Wollaston, and Ritter, respecting the invisible rays of the spectrum; but it does not appear to be easily reconciled to any hypothesis which we are able to frame respecting the nature of vision.

The uniform velocity of light is, on the other hand, an exact and immediate consequence of the principles of the wave-theory. It follows from these principles, that the velocity with which vibratory movement is propagated in an elastic medium depends in no degree on the exciting cause, but varies solely with the elasticity of the medium and its

density. If these then be supposed to be *uniform* in the vast spaces which intervene between the material bodies of the universe, (and it is hard to suppose it otherwise,) the velocity will be the same, whatever be the luminous origin.

The *rectilinear motion* of light has long been urged in favour of the theory of emission, and against the wave-theory. If light consists in the undulations of an ethereal medium, (it has been said) as sound consists in the undulations of the air, it should be propagated in all directions from every new centre, and so bend round interposed obstacles. Thus luminous objects should be visible, even when an obstacle is between them and the eye, (just as sounding bodies are heard, though a dense body may be interposed between them and the ear,) and *shadows* could not exist.

To this objection (which was that chiefly urged by Newton himself) it might be enough to reply, that though vibratory motion in an elastic medium is propagated in *all* directions from every new centre, yet we have no reason to assume that it is propagated with the *same intensity* in every direction, however inclined to that of the original wave. Judging from analogy, we should rather say that there is every reason for an opposite conclusion; for there are a multitude of facts which prove that sound is *not* propagated with the same intensity in all directions, however inclined to the direction of the original motion. Now, if there be *any* difference between the intensity of the direct and lateral propagation, we may suppose this difference to be ever so great; i. e. we may suppose the ethereal medium to be so constituted that the intensity of the laterally propagated vibration shall be insensible.

But the solution of the difficulty rests upon more solid grounds than loose analogy. We can apply the strictest mathematical calculation to the problem, and are able to show that the observed facts are in complete accordance with the deductions of *a priori* reasoning. Before we enter

on this subject, however, it will be necessary to consider, somewhat more minutely, the nature of a wave, and its mode of propagation.

Let us conceive, then, a cord stretched in a horizontal position, one end being attached to a fixed point, and the other held in the hand. If the latter extremity be agitated, by the motion of the hand up and down, a series of waves will be propagated along the cord, each of which will advance uniformly. Here it is evident that each particle of the cord has merely a vibratory motion in a vertical direction. But as this vibratory motion is communicated from one particle to the next, from that again to the succeeding,—and so on, along the whole length of the cord,—it will follow that some of the particles will reach their highest position, when others are in the lowest; and that other particles, intermediate to these, are neither in their highest or their lowest position, but in some intermediate stage of their vibration. Thus, while each particle moves only to and fro vertically, an undulation or wave will be propagated horizontally along the string; and there will be a succession of similar undulations as long as the original disturbance continues. The particles a, a', a'' , (fig. 1.) or the particles b, b', b'' , &c. are said to be in similar *phases* of vibration. The *wave*, or undulation, consists of all the particles between two which are in similar phases,—as between a and a' , or between b and b' ; and the *length of a wave* is the distance between them estimated in the direction in which the motion is propagated. It is evident from this description that a wave, or undulation, comprises particles in every phase of their vibration.

Now, instead of a single string, let us suppose an infinite number, all diverging from the same centre; and let us suppose that they are each caused to undulate by a disturbing action at that centre, acting in a similar manner, and in the same degree, on all. It is obvious then that an undulation will be propagated along all the strings, and that these un-

dulations will be equal in magnitude, and will be propagated with the same velocity, provided the strings be equal in tension, elasticity, and other respects. In this case, then, similar waves will be propagated to points equally distant from the origin of disturbance in the same time; and all the points which are in a similar phase of vibration will be situated on the surface of a *sphere*, of which that origin is the centre.

Now, in the place of the actual strings we have been considering, let us imagine rows of ethereal particles connected by their mutual actions, and the whole of what we have said will apply to the propagation of light, the luminous body being the source of disturbance. We can easily understand, then, how in the same medium an undulation may be propagated spherically from the luminous centre. The length of the wave is the distance, estimated in any direction from the centre, of two particles which are in similar phases of vibration; and it is therefore the space through which the vibratory movement is propagated in the time of a single vibration. Accordingly if λ denote the length of the wave, τ the time of vibration, and v the velocity of wave-propagation, $\lambda = v\tau$.

We have hitherto considered the propagation of vibratory movement without reference to any diversity in its nature. It is obvious, however, that vibrations of any kind may differ from one another in two particulars,—namely, in the *space of vibration*, and in the *time*. In the aerial pulses the amplitude of the vibration determines the *loudness* of the sound; and the frequency of the pulses, or the time of vibration, determines its *note*. In like manner the amplitude of the *ethereal* vibrations determines the *intensity of the light*; and their frequency, or the period of vibration, determines the *colour*. Thus, two lights may differ from one another in intensity and colour, the former depending (according to the wave-theory) on the space of vibration, and the latter on the time.

But though the intensity of the light is obviously dependent on the amplitude of the vibration, yet it does not appear, *a priori*, by what power of the amplitude it is to be represented. In fact, we must *define* what we mean by a *double, triple, &c.* quantity of light, before we can know how that quantity is to be mathematically measured. If then we say that a *double light* is the *sum* of the lights produced by *two* luminous origins of equal intensity, placed close together, it is easy to prove that the quantity of light, in general, is measured by the *square* of the amplitude of the vibration. From this it follows that the intensity of the light diverging from any luminous origin must decrease inversely as the square of the distance; for, from the laws of wave-propagation it appears that the space of vibration diminishes in the inverse simple ratio of the distance. Thus the known law of the variation of the intensity of light is deduced from the principles of undulatory propagation.

The *colour* of the light (we have said) depends on the number of impulses which the nerves of the eye receive, in a given time, from the vibrating particles of the ether;—the sensation of *violet* colour being produced by the most frequent vibrations, and that of *red* by the least frequent. Now the number of vibrations performed in a given time varies inversely as the time of a single vibration. The colour of the light, therefore, varies with the time of vibration, or with the *length of the wave*. By experiments, which will be described hereafter, it has been found that the length of a wave corresponding to the *extreme red* of the spectrum is 266 ten-millionths of an inch, and that corresponding to the *extreme violet* 167 ten-millionths. The length of the wave corresponding to the ray of mean refrangibility is nearly 200 ten-millionths, or $\frac{1}{5000000}$ th of an inch. It appears, then, that the sensibility of the eye is confined within much narrower limits than that of the ear; the ratio of the times of the extreme vibrations which affect the eye being only

that of 1.58 to 1, which is less than the ratio of the times of vibration of a fundamental note and its octave. There is no reason for supposing, however, that the vibrations themselves are confined within these limits. In fact, we know that there are *invisible* rays *beyond* the two extremities of the spectrum, whose periods of vibration (and length of wave) must fall without the limits now stated to belong to the visible rays.

Analysis seems to indicate that the velocity of wave-propagation is always the same in the same homogeneous medium, and is independent both of the space of vibration and of the time. Thus all sounds, whether loud or low, grave or acute, travel in the same medium with the same velocity; and it would seem, in like manner, that light, if it consisted in the undulations of an ethereal medium, should be propagated with the same velocity, whatever be its intensity or colour. We shall presently see that this conclusion must be restricted.

Let us now proceed to consider the phenomena which take place when light encounters a new medium. In this case, you are all aware, it is either *bent back* into the medium in which it has been moving, the incident and reflected rays making equal angles with the surface;—or else it *enters* the new medium, though in an *altered direction*, the cosines of the inclinations of the two portions of the ray to the bounding surface being in an invariable ratio.

To account for these phenomena of reflexion and refraction, it is supposed, in the Newtonian theory, that the particles of bodies and those of light exert a *mutual action*;—that, when they are nearly in contact, this action is *attractive*;—that, at a distance a little greater, the attractive force is changed into a *repulsive* one;—and that the attractive and repulsive forces succeed one another, probably for many alternations. Nothing can be more reasonable than this hypothesis, granting that light is material; for the succession

of attractive and repulsive forces, here assumed, is altogether similar to that to which the phenomena of molecular action are ascribed. On these suppositions Newton has rigorously deduced the laws of reflexion and refraction. It is shown, in the former case, that the whole perpendicular velocity of the molecule is restored to it in an opposite direction, by the operation of the repulsive force; and therefore, that the angles which its path makes with the perpendicular to the surface, before and after reflexion, are equal. In the latter case it is proved that the effect of the attractive force is to increase the perpendicular velocity of the molecule, and by an amount which is constant for the same medium; from which it follows, that the sines of the angles which the course of the ray makes with the perpendicular to the surface, before and after refraction, are in an invariable ratio,—that ratio being the inverse of the ratio of the velocities in the two media. I will not detain you further on the steps of this deduction, as it is to be found in the *Principia*, and in other works in the hands of all. Suffice it to say, that the problem is the first in which the effects of that important class of forces, acting only at insensible distances, has been submitted to calculation; and that its solution is regarded by M. Poisson as forming an era in the history of science.

But though the theory of emission is perfectly successful in explaining the laws of reflexion and refraction, considered as distinct phenomena, yet it is by no means equally so in accounting for their connexion and mutual dependence. When a beam of light is incident on the surface of any transparent medium, part is, in all cases, transmitted, and part reflected; the intensity of the reflexion being less, the less the difference of the refractive indices of the two media, and the reflexion ceasing altogether when this difference vanishes. How is it then that some of the molecules obey the influence of the repulsive force, and are reflected; while others

yield to the attractive force, and are refracted? To account for this Newton was obliged to have recourse to a new hypothesis. The molecules of light, in their progress through space, are supposed to pass continually into two alternate states, or fits, which recur periodically and at equal intervals. While in one of these states, called the *fit of easy reflexion*, they are disposed to obey the repulsive or reflective forces of the body which they meet; and, on the other hand, they yield more readily to the attractive or refractive forces, when in the alternate state, or *fit of easy transmission*. Now the molecules composing a beam of common light are supposed to be in every possible stage of these fits, when they reach the surface;—some of them in a fit of reflexion, and others in a fit of transmission. Some of them, consequently, will be reflected and others refracted, and the proportion of the former to the latter will increase with the incidence.

To account for the fits themselves, Newton assumed the existence of an ethereal medium, analogous to that of Huygens, although he did not assign it the same office. The molecules of light were supposed to excite the vibrations of this ether, just as a stone flung into water raises waves on its surface. This vibratory motion was supposed to be propagated with a velocity greater than that of the molecules; so as to overtake them, and impress upon them the disposition in question by conspiring with or opposing their progressive motion. In one of his queries Newton has even calculated the elastic force of this ether, as compared with that of air, in order that the velocity of propagation should exceed that of light.

I need not dwell upon the physical account of the fits given by Boscovich, or that suggested by Mr. Melville and maintained by M. Biot. The theory itself has now lost much of its credit, since the phenomena of the colours of thin plates (phenomena which first suggested it to the mind of Newton) have been shown to be irreconcilable with it. The expla-

nation which it yields of the facts now under consideration is alike inconsistent with phenomena. In fact, the molecules which are transmitted, are not all in the *maximum* of the fit of transmission; but are supposed to reach the surface in every possible stage, or *phase*, of this, which may be called the *positive* fit. But as a change of the fit from positive to negative is, in general, sufficient to overcome altogether the effect of the attractive force, and subject the molecule to the repulsive, it is obvious that the phase of the fit must modify the effects of these forces in every intermediate degree; and that the molecules which do obey the attractive force, must have their velocities augmented in different degrees, depending on their phase. Hence, as the direction of the refracted ray depends on its velocity, the transmitted beam should consist of rays refracted in widely different angles, and should be scattered and irregular.

Let us now turn to the account which the other theory gives of the same phenomena and of their laws.

The velocity of propagation, in the wave-theory of light, depends on the elasticity of the vibrating medium as compared with its density. In the same homogeneous medium the velocity will be therefore *constant*, and the wave propagated from any centre of disturbance *spherical*. But when a wave reaches the surface of a new medium whose elasticity is different, it will give rise to two waves, one in each medium; and both differing in position from the original wave. In fact, it is obvious that, in general, the several portions of the incident wave will reach the bounding surface at different moments of time. *Each* of these portions will be the centre of two new waves, one of which will be propagated in the first medium with the original velocity, while the other will be propagated in the new medium, and with the velocity which belongs to it; so that there will be an infinite number of *partial waves* in both media, diverging from the several points of the bounding surface. But, by the

principle of the coexistence of small motions, the agitation of any particle of either medium is the sum of the agitations sent there at the same instant from these several centres of disturbance. The surfaces on which these are accumulated will be the reflected and refracted waves, and they are obviously those which *touch* all the small spherical waves at any instant.

Thus, let mn (fig. 2.) be the front of a *plane* wave, meeting the reflecting surface at m . Each portion of this wave, as it reaches the surface, becomes the centre of a diverging spherical wave, which will be propagated with the velocity of the original wave. Accordingly, when the portion n reaches the surface at k , the portion m will have diverged into the spherical wave whose radius, mo , is equal to nk . And, in like manner, if $m'n'$ be drawn parallel to mn , the wave diverging from m' will in the same time have reached the spherical surface whose radius, $m'o'$, is equal to $n'k$. The surface which touches all these spheres at any instant is that of the reflected wave. But, as mo and $m'o'$ are proportional to mk and $m'k$, it is obvious that this tangent surface is plane; and since $mo = nk$, and the angles at n and o are right, it follows that the angles nmk and okm are equal,—or that the incident and reflected waves are equally inclined to the reflecting surface.

The proof of the law of refraction is in all respects analogous to the preceding. Let mn (fig. 3.) be the position of the incident plane wave at any moment. In an interval of time proportional to nk , the portion n of this wave will have reached the surface at k , and the portions m and m' will have become the centres of diverging spherical waves in the second medium,—the radii of these spheres, mo and $m'o'$, being to the intercepts, nk and $n'k$, in the constant ratio of the velocities of propagation in the two media. The surface which touches these spheres is that of the refracted wave. It is

obvious, as before, that it is plane; and, since $\sin nmk : \sin mko :: nk : mo$, we learn that the sines of the angles which the incident and refracted waves make with the refracting surface, are in the constant ratio of the velocities of propagation.

Such is the demonstration of the laws of reflexion and refraction given by Huygens. The composition of the *grand* or primary wave, by the union of the several secondary or *partial* waves, in this demonstration, has been denominated the *principle of Huygens*; and it is obviously a case of the more general principle of the coexistence of small motions. It easily follows from this mode of composition that the surface of the primary wave marks the extreme limits to which the vibratory movement is propagated in any direction in any given time; so that light is propagated from any one point to another in the *least possible time*. This is the well known law of Fermat,—the law of swiftest propagation; and it will appear from what has been stated, that it holds whatever be the number of modifications which the course of the light may undergo by reflexion or refraction.

This law may be thus enunciated—"The course pursued by any reflected or refracted ray is that which would be described in the least possible time, by a body moving from any point on the incident to any point on the reflected or refracted ray." If l therefore denote the length of the path described by the incident light, between any assumed point and the point of incidence, l' the corresponding length of the path described by the reflected or refracted light, and v and v' the velocities of propagation, the sum of the times, $\frac{l}{v} + \frac{l'}{v'}$ is a minimum; or, multiplying by v , and denoting the ratio, $\frac{v}{v'}$, by μ ,

$$l + \mu l' = \text{minimum.}$$

In the case of reflexion, $\mu = 1$, and $l + l'$ is a minimum. The course pursued by a reflected ray is therefore such that the sum of the paths described between any two points and the reflecting surface is the least possible. The constant factor, μ , in the case of refracted light, is the refractive index of the medium.

The intensity of the light in the reflected and refracted waves will depend on the relative densities of the ether in the two media. For we may compare the contiguous strata of ether in these media to two elastic bodies of different masses, one of which moves the other by impact; and it is easy to deduce, on this principle, the intensities of the reflected and refracted light in the case of perpendicular incidence.

On reviewing what has been said, we cannot but be struck by the remarkable fact, that theories so widely opposed as the theory of emission and that of waves should lead mathematically to the same result. According to both, we have seen, the ratio of the sines of incidence and refraction is equal to the ratio of the velocities of light in the two media, and therefore constant. But there is this important difference between them: in the wave-theory, the sines of these angles are in the *direct* ratio of the velocities, while, according to the theory of emission, they must be in the *inverse*. In other words, the velocity of light in the denser medium must be *less* according to the principles of the former theory; while, according to the latter, it is *greater*. Here then the rival theories are directly at issue upon a point of fact, and we have only to ascertain how this fact stands, in order to be able to decide between them. The important experiment by which this is accomplished was made by M. Arago; and the result, as we shall see in our next lecture, is conclusive in favour of the wave-theory.

It remains to say a few words of the *dispersion* of light.

To account for this phenomenon, the modern advocates of the theory of emission have been forced to assume that the molecules of light are *heterogeneous*, and that the attractions exerted on them by bodies vary with their nature, being in this respect analogous to chemical affinities. This supposition, as Dr. Young has justly observed, is but veiling our inability to assign a *mechanical* cause for the phenomenon.

According to the principles of the wave-theory, the *colour* of light is determined by the frequency of the ethereal vibrations, or by the length of the wave;—the longest waves producing the sensation of *red*, and the shortest that of *violet*. Now observation proves that the refractive index, (or the ratio of the velocities of propagation in the two media,) is different for the light of different colours. The advocates of the wave-theory are therefore forced to conclude that the velocity of propagation in a refracting medium *varies with the length of the wave*. Here, then, we encounter a difficulty in this theory, which has long been regarded as the most formidable obstacle to its reception. Theory seemed to indicate that the velocity of wave-propagation was *constant* in the same medium, and depended solely on the elasticity of the medium as compared with its density. That velocity, therefore, should be the same for light of all colours, as it is for sound of all notes; and all rays should be equally refracted.

I will not dwell on the various attempts which have been made to account for this apparent discrepancy of observation and theory. The difficulty appears at length to have been overcome by the analytical skill of M. Cauchy. In fact, the conclusion of analysis to which we have just adverted,—namely, that the velocity of wave-propagation is constant in the same homogeneous medium,—is deduced on the particular supposition that the sphere of action of the molecules of the vi-

brating medium is indefinitely small compared with the length of a wave. This restriction, you will observe, is but a mathematical fiction, adopted for the purpose of approximating to the solution of a problem, which, in its entire generality, seemed to defy the powers of analysis. If this restriction be removed, we have no longer any ground for concluding that the waves of different lengths will be propagated with the same velocity; and the mathematical conclusion hitherto acquiesced in must be regarded but as the approximate result of an incomplete analysis. It was in this point of view that the problem presented itself to M. Cauchy. In the profound researches of this mathematician relating to light, the ether is considered as a system of particles solicited by mutual attractions and repulsions; and from the partial differential equations which represent their movement, he had already deduced the laws of propagation in crystallized, as well as homogeneous media. These equations, however, were but approximate, and derived from others of greater generality by the omission of certain terms, which it was supposed could not influence the result. Resuming the problem of wave-propagation with the more general equations, he has at length arrived at the desired conclusion,—that there exists, generally, a relation between the *velocity of propagation* (or the refractive index) and the *length of the wave*; and, therefore, that the rays of different colours will be differently refracted. The relation to which I have alluded is expressed by an equation which involves two arbitrary constants depending on the nature of the medium. These constants will be determined, therefore, when we know two values of the refractive index corresponding to waves of known length; and the equation may thenceforward be applied to compute the refractive index in the same medium corresponding to any other assumed value of the length of the wave. This has been done by

Professor Powell for a great number of media, and the deduced values of the refractive index have been compared with those observed by the accurate method of Fraunhofer. The differences do not appear to surpass the limits of the errors of observation.

LECTURE II.

INTERFERENCE OF LIGHT.

GENTLEMEN,—In our last lecture we considered the mode of propagation of a luminous wave, and the modifications which it underwent on encountering the surface of a new medium. We may now proceed to inquire what will be the effect, when two series of waves are propagated simultaneously from two near luminous origins.

It is obvious that when two waves,—one proceeding from each origin,—arrive at any instant at the same point of space, the particle of ether there will be thrown into vibration by both; and we are to consider what will be the result of this double vibration. Now, it is demonstrated by analysis, that when *two small* vibrations are communicated at the same time to a material point, each of them will subsist independently of the other; and the motion of the point will in consequence be the *resultant* of the motions due to each vibration considered separately. This principle is denominated the *superposition of small motions*. Its truth may be made clear by a simple instance. Let a pendulous body receive an impulse in any plane passing through the point of suspension. It will then, of course, vibrate in that plane. Now, at the lowest point of the arc of vibration, let a second impulse be given to the moving body in a direction perpendicular to the plane in

which it already vibrates. This impulse, if communicated to the body at rest, would cause it to vibrate in a plane at right angles to the former, and through an arc depending on the magnitude of the impulse. Now it will be found on trial that the distance of the body from the vertical, measured in either of these planes, is the same at any instant as if the other vibration did not exist; so that each vibration subsists *independently* of the other, and the result will be a compound elliptical vibration. We have here supposed the coexisting vibrations to take place in separate planes, in order that their independence may be more distinctly recognized. When the two vibrations are in the *same plane*, it is obvious that the resulting vibration will be also in that plane; and that its amplitude will be the *sum* of the amplitudes of the component vibrations when their directions conspire, but their *difference* when they are opposed.

Let us transfer this to the case of light:—Let us suppose that two sets of waves start at the same time from two near luminous origins, (which, for simplicity, we shall assume to be of equal intensity,) and that a distant particle of ether is thrown into vibration by both at the same time. Then, supposing that these two vibrations are performed in the same plane, it follows from what has been said, that, when their directions conspire, they will be added together, and the resulting space of vibration will be *double* of either; and, on the contrary, they will counteract one another, and the resulting vibration will be reduced to *nothing*, when their directions are opposed.

It is evident, further, that the *directions* of the vibrations will *conspire*, and therefore the space of vibration be doubled, when the two waves arrive in the *same phase*; and that, on the contrary, their directions will be opposed, and the resulting vibration reduced to nothing, when they arrive in *opposite phases*. Let the waving lines AB and A'B', or AB and

$A''B''$ (fig. 4.) represent the two undulations; the distance of any particle from its state of rest being represented by the ordinate, or perpendicular, at the corresponding point of the horizontal or *mean* line. Then, if the undulation $A'B'$ be superposed upon AB , the corresponding points of each being in the *same phase*, it is evident that the distances by which the particle at any point is removed from its state of rest by each, mn and $m'n'$, will be added together, and the space of vibration *doubled*. Whereas, if the undulations $A''B''$ and AB , whose corresponding points are in *opposite phases*, be superposed, the distances from the position of rest, mn and $m'n''$, lie on opposite sides of the mean line, and when added together *destroy one another*. Thus the space of vibration is doubled when the waves arrive at the same point in the same phase: it is annihilated, when they arrive in opposite phases. Now the *intensity of the light* is as the square of the space of vibration. The intensity therefore is *quadrupled* in the former case, and *destroyed* in the latter.

We have here taken, for the sake of illustration, two of the most important cases,—those, namely, in which the co-existing undulations are in complete *accordance*, or complete *discordance*. When this is not the case, and the waves meet in some intermediate stage of the vibratory movement, the *position* of the maximum vibration will be altered as well as its *magnitude*; and the rules for their composition bear a close analogy to the well known rule for the composition of forces.

We learn, then, as a result of the wave-theory, that two lights may either augment each other's effects; or they may partially, or even wholly, destroy one another, and thus, by their union, produce *complete darkness*.

Before we proceed to examine more particularly this indication of theory, we may observe that it is altogether analogous to what is known to take place in other cases of vibratory motion. If two waves of water arrive at the same

point at the same instant, in such a manner that the crest of one wave coincides with that of the other, their effects will be added together, and the water at that point will be raised into a wave whose height is the sum of the heights of the conspiring waves. If, on the other hand, the crest of one wave coincides with the *sinus* or depression of the other, the height of the resultant wave will be the difference of the components; and when these are equal, the resultant wave will entirely disappear.

We have a magnificent example of these effects in the well known phenomena of the *spring* and *neap* tides; the tidal wave in the former case being the sum of the waves caused by the action of the sun and moon, and in the latter their difference. The peculiarity of the tides in the port of Batsha furnishes a still more striking instance of the principle of interference. The tidal wave reaches this port by two distinct channels, which are so unequal in length that the time of arrival by one passage is exactly six hours longer than by the other. It follows from this that when the crest of the tidal wave, or the *high water*, reaches the port by one channel, it is met by the depression, or *low water*, coming through the other; and when these opposite effects are also equal, they completely neutralize each other. At particular seasons, therefore, when the morning and evening tides are equal, there is *no tide* whatever in the port of Batsha; while at other seasons there is but *one* tide in the day, whose height is only the difference of the heights of the usual morning and evening tides. The disappearance of the diurnal inequality in the port of London has been recently shown by Mr. Whewell to depend on a similar interference of two tide-waves, one of which reaches this port by the English channel, while the other arrives there twelve hours later, having taken the circuit of the northern coast.

Analogous phenomena take place in sound, and produce the coincidences or *beats* in music. You will easily un-

derstand, from what has been said, that these effects occur when the condensed part of the aerial pulse arising from one origin of sound coincides with the rarified part of that proceeding from the other. They are often heard during the playing of a large organ, and give rise to the swelling and falling sounds which we hear, especially among the lower notes of the instrument.

The interference of the aerial pulses may be exhibited in a more instructive form. Let a compound tube be taken, consisting of two equal and similar branches terminating in a common trunk. It is evident, then, that if the air be thrown into the *same* state of vibration at the extremities of the two branches,—the particles going and returning simultaneously in both,—a *double* vibration will be propagated to the extremity of the main trunk, and may be rendered sensible by the agitation of the particles of sand on a stretched membrane. If, on the other hand, the air be in *opposite* states of vibration at the extremities of the branches, these will neutralize one another in the trunk, and the membrane and the sand will be quiescent. The conditions here described are attained by bringing the ends of the branches over the parts of a vibrating plate which are in similar, or in opposite states of vibration; and the result is satisfactory and conclusive. When the length of the tube is such that it is *in unison* with the vibrating plate, it will utter a distinct sound in the one case, while in the other it will be silent. This ingenious experiment was devised by Mr. Hopkins. The alternate augmentation and intermission of sound observed by Dr. Young, when a tuning fork is turned round its axis at a short distance from the ear, are easily referred to the same principles.

That *two lights*, then, should *produce darkness*, is a result of the same kind as that two sounds should cause *silence*, or that two waves should make a dead level. But we are not left to analogy alone for the proof of this remarkable conse-

quence of the wave-theory of light. The phenomenon itself has been established by the most direct and convincing experiments, and we shall soon see that it is observed in a multitude of cases where its existence was at first little suspected.

This important law,—well known under the name of the *interference of light*,—was for the first time distinctly stated and established by Dr. Thomas Young, although some facts connected with it were known to Grimaldi. The latter writer had even explicitly asserted that “an illuminated body may be rendered darker by the *addition of light*,” and adduced a simple experiment in proof of it. Grimaldi’s experiment was as follows. Let the sun’s light be admitted into a darkened chamber through two small and equal apertures of a circular form. Two diverging cones of light will be thus produced; and each of these cones will be surrounded by a penumbra in which the illumination is only partial. Now let these two beams be received on a screen at some distance, where the penumbras of the two cones overlap. It will be then observed, that although the greater part of this doubly illuminated space is brighter than the penumbra of one cone alone, yet the boundaries of the overlapping portions are much darker than the other parts of the penumbras which do not overlap; and if one of the beams be intercepted by an obstacle, this dark part will recover the brightness of the rest. Thus darkness may be produced by *adding light*; and, on the other hand, by withdrawing a portion of the light we may augment the illumination.

This interesting experiment assumed a more distinct and decisive character in the hands of Dr. Young. If the two apertures, in Grimaldi’s experiment, be reduced to a very small size, and brought close together, and if the original light be homogeneous, we shall observe a series of alternate *bright* and *dark* bands, formed at those points where the waves proceeding from the two origins conspire or are

opposed. That these alternations of light and darkness are caused by the mutual action of the two beams, is proved by the fact,—that if one of the beams be intercepted by closing one of the apertures, the whole system of bands will disappear, and the light which remains become of uniform intensity. By withdrawing one of the lights, then, the dark intervals recover their brightness and become luminous; so that darkness, in this case, must have been produced by the action of one light on the other.

We shall best understand the circumstances of this phenomenon, by considering what takes place in another more familiar case of interference. If two stones be flung at the same instant into a pool of stagnant water, a series of circular waves will be propagated from each of the two centres of disturbance; and where these two sets of waves *cross*, they will produce effects similar to those we have been describing in the case of light. Where the *crest* of one wave falls upon the crest of another, they will be added together, and form a higher crest, or *ridge*, on the surface. And, on the contrary, where the crest of one wave meets the hollow, or *sinus*, of another, they will counteract one another's effects, and the water will stand at that point at its original level, as if undisturbed. It is obvious that there will be several sets of consecutive points of each class, or several lines of *double disturbance* and *no disturbance*. One line of double disturbance, AA, (fig. 5.) will be produced by the meeting of waves *equidistant from the two centres*;—as the first of one with the first of the other, the second of one with the second of the other, &c. This line is necessarily *straight*. On either side of this there will be a line, BB, B'B', consisting of those points where the first wave from one origin encounters the second from the other, the second from one the third from the other,—of all those points, in short, whose distances from the two centres *differ by the length of a single wave*. The next pair of lines, CC, C'C', consist of those points

whose distances from the two centres differ by the *length of two waves*; and so on. All these lines are hyperbolas, and on all of them the disturbance is doubled, and an elevated ridge raised on the surface. But there are likewise intermediate lines composed of those points whose distances from the two centres differ by *half a wave*, by a *wave and half*, by *two waves and half*, &c. On all these lines, the crest of the wave from one origin meets the sinus of a wave from the other; and these, therefore, are the lines of *no disturbance*. They are evidently hyperbolas like the former.

All that has been now said applies strictly to the phenomena of light, in the aspect under which they are presented by the wave-theory. It has been already shown to be a consequence of that theory, that when two luminous waves of equal intensity arrive at the same point in the *same phase* of vibration, the intensity of the light will be *quadrupled*;—that they will destroy one another's effects, and produce *darkness*, when they arrive in *opposite phases*;—and, finally, that the intensity of the resulting light will be intermediate to these two extremes, when the waves meet in some intermediate stage of the vibratory movement. It is easy to understand, therefore, in what manner the alternate bright and dark bands are accounted for. In the same medium the waves of any given length are propagated with a constant velocity. When therefore two series of waves of equal length diverge at the same time from two centres, they will arrive at the same point in the *same phase*, provided that the lengths of the paths traversed are *equal*, or differ by any *whole number of undulations*. They meet in *opposite phases*, on the other hand, when the lengths of their paths differ by *half a wave*, or by any *odd multiple* of half a wave. The central bright band, then, is formed at those points where the distances traversed are *equal*. The next bright band on either side is produced where the distances traversed differ by the length of *one entire wave*. The succeeding pair

where the distances differ by *two* whole waves; and so on. In the same manner, the first *dark* band is produced on either side of the central bright one, and at points for which the distances traversed differ by the length of *half a wave*. The second pair of dark bands where these distances differ by *one wave and half*; and so for the others.

Thus, in Young's experiment, if the light diverging from the two apertures o , and o' , (fig. 6) be received on a screen, AD , it is found that the central bright band is formed at the point A , where the screen is intersected by the line PA , which bisects the line oo' , and is perpendicular to it. The *central* band, therefore, is formed where the paths traversed by the two pencils are *equal*. There will be a dark band on either side of the central bright one, and, beyond these, a pair of bright bands. If we measure the distances of one of these from the two apertures, we shall find that their difference, $bo' - bo$, is a *constant* quantity, whatever be the position of the screen. This difference is the *length of a wave*. Beyond these is a second pair of bright bands, the difference of whose distances from the two centres, $co' - co$, is double of the preceding, or equal to *two* whole waves; and in like manner, the difference of the lengths of the paths, at the place of each succeeding bright band, is found to be some exact multiple of the first difference, or of the length of a wave.

Performing the same measurements for the intermediate dark bands, we find that the difference in the lengths of the paths, where the first pair is formed, is half the difference, $bo' - bo$, or *half the length of a wave*. The differences of the paths, at the place of each succeeding pair of dark bands, are found in like manner to be intermediate to the corresponding differences for the bright bands on either side, or to be equal to some *odd* multiple of half a wave.

The difference of the distances from the two apertures being constant for the successive points of the same band, it follows that these points must form an hyperbola, whose foci

coincide with the two apertures. It will be easily seen that the curvature of these hyperbolic lines is very small, except close to their vertices; and that we may, without sensible error, consider them as coincident with their asymptots.

It is easy to determine the position of the bands, as dependent on the interval of the apertures, and on the distance of the screen.

The place of any bright or dark band, m , (fig. 7.) is determined by the condition that the difference of its distances from the two apertures, $mo' - mo$, is an integer multiple of the length of half a wave. Now, drawing the lines on , $o'n'$, parallel to PA , and denoting the distance AP by b , the interval of the apertures, oo' , by c , and the distance Λm by x , the right-angled triangles omn , $o'mn'$, give

$$om = \sqrt{b^2 + (x - \frac{1}{2}c)^2} = b + \frac{(x - \frac{1}{2}c)^2}{2b},$$

$$o'm = \sqrt{b^2 + (x + \frac{1}{2}c)^2} = b + \frac{(x + \frac{1}{2}c)^2}{2b};$$

the distance b being very great in comparison with x and c . Hence

$$o'm - om = \frac{(x + \frac{1}{2}c)^2 - (x - \frac{1}{2}c)^2}{2b} = \frac{cx}{b}.$$

But this difference is equal to $n \frac{\lambda}{2}$, λ being the length of a wave; we have, therefore,

$$x = \frac{nb\lambda}{2c};$$

in which the even values of n correspond to the places of the bright bands, and the odd values to those of the dark ones.

The preceding formula enables us to compute the length of a wave of light, when the distances b , c , and x , have been determined by accurate measurement. It has been found in this manner that the length of a wave is .0000266 of an inch for the extreme red rays; .0000167 for the extreme violet;

and .0000203, or about the $\frac{1}{50000}$ of an inch, for the mean rays of the spectrum.

But though the principle of interference seemed to be established by the experiments and reasonings of Young, yet it was not freed from all question. It might be supposed that the light passing by the edges of the apertures, in the experiment last described, underwent modifications of some kind or other which produced the observed effects. It was therefore of importance to show that these effects were wholly independent of *apertures* or *edges*; and that *any two rays* proceeding from the same luminous origin, and meeting under a small obliquity, will *interfere* in the manner already described, whatever be the attending circumstances. This has been done by Fresnel; and the experiment which he devised for the purpose has been justly ranked among the most important and instructive in the whole range of physical optics.

Two plane mirrors are placed so as to meet at a very obtuse angle. A beam of light diverging from the focus of a lens is suffered to fall upon them; and there will be therefore two reflected beams, whose directions are inclined at a very small angle. Here then are two beams diverging from the same luminous origin, and separated simply by reflexion at plane surfaces, without the intervention of edges, or of any thing accidental which can be regarded as contributing to the result. These beams, however, still interfere, and produce a succession of alternate bright and dark bands, in the manner already explained. In order to satisfy ourselves that these bands are in fact produced by the mutual action of the two beams, we have only to intercept one of them, by covering one of the mirrors, and the whole system instantly vanishes.

Let qn and qn' (fig. 8) represent the sections of the two mirrors, which we shall suppose to be perpendicular to the plane of the diagram; and let k be the luminous origin, or

the focus of the lens in which the sun's rays are concentrated. Then taking the points o and o' at equal distances on opposite sides of the mirrors, these points will be the foci of the two reflected pencils, or the points of divergence of the two beams. Now it is found, in the first place, that the bands are parallel to the line of intersection of the two mirrors; secondly, that they are symmetrically placed on either side of a plane passing through this intersection, and through the point p , which bisects the interval between the two foci o and o' : and thirdly, that in proceeding from the mirrors, they are propagated in hyperbolas, whose foci are o and o' , and whose common centre is p .

These results are in exact accordance with theory. In fact, since $on = nk$, and $o'n' = n'k$, the difference of the paths traversed by the reflected rays, knm and $kn'm$, when they meet at m , is the same as if they had reached that point, diverging directly from the points o and o' . All, then, that we have said respecting the interference of the pencils diverging from two near luminous origins, will apply to this case. Since $kq = oq = o'q$, the line qp , which bisects the interval of the foci oo' , is also perpendicular to it, and any point of it, as Λ , is equidistant from these two centres. The bands, therefore, are symmetrically situated with respect to this line; and the distance, Λm , of the band of any order from the central band, is equal to $\frac{n\lambda_{\Lambda p}}{oo'}$.

This distance is easily expressed in terms of given quantities. For $pq = oq \cdot \cos oqp = kq \cdot \cos oqp$; and $oo' = 2op = 2kq \cdot \sin oqp$. But since the angles kqo , kqo' , are bisected by the lines qn and qn' , it is easy to see that the angle oqp (or the half of the angle oqo') is equal to the inclination of the mirrors. If then this inclination be denoted by ϵ , and the distances kq and qa by a and b , we have

$$oo' = 2a \sin \epsilon, \quad \Lambda p = a \cos \epsilon + b = a + b; \quad q.p.$$

and therefore the distance of the band of the n^{th} order from the centre is expressed by the formula

$$\frac{(a+b)n\lambda}{2a \sin \epsilon}.$$

I have found that the phenomenon of interference is displayed in a striking manner by the mutual action of *direct* and *reflected* light; and the experiment in this form is much more manageable than that of Fresnel. We have only to take a single piece of plate glass, or a metallic reflector, and place it in such a position that the rays diverging from the luminous origin shall be reflected at an angle of nearly 90° . A screen placed on the other side of the mirror will receive both the direct and reflected pencils; and as they meet under a small angle, and have traversed paths differing by a small amount, they are in a condition to interfere. It will be readily seen that the system of bands formed in this manner is but half of that produced in Fresnel's experiment. There are some other peculiarities of this system, into which time will not permit me to enter, which seem to illustrate certain abstruse points of the wave-theory.

There is yet another mode of studying the fundamental phenomenon of interference, which is in some respects more convenient than any of the former. It is obvious that the original beam may be separated by *refraction*, as well as *reflection*; and if the inclination of the two refracted pencils be small, similar results will take place. For this purpose it is only necessary to procure a prism with a very obtuse angle, and to allow the beam of light to fall perpendicularly on the opposite face. It is evident then that this beam will be differently refracted, at emergence, by the two faces which contain the obtuse angle; and that it will be thus divided into two beams, which will be slightly inclined. These beams then proceed from one common origin, and meet under a small obliquity, and therefore fulfil all the conditions ne-

cessary for their interference. It is found, accordingly, that a series of alternate bright and dark bands is formed parallel to the edge of the prism.

It will be evident, from what has been said, that the central fringe produced by the interference of two pencils, is the locus of those points at which they arrive in the *same time*; and, accordingly, when neither of the pencils has experienced any interruption in its progress, the points of that fringe will be equally distant from the two luminous origins. The case is altered, however, if we interpose a thin plate of a denser medium in the path of one of the interfering rays. If the light is *retarded* in the denser medium, it is obvious that the points reached in the same time will no longer be equally distant from the two centres, but will be *nearer* to that whose light has undergone the retardation. The reverse will be the case if the light is *accelerated* in the interposed plate; so that the central fringe, and the whole system, will be shifted *towards* the side of the interposed plate in the former case, and *from* it in the latter. Here then we have a complete *experimentum crucis*, by which to decide between the theory of emission and that of waves; and its result, as we have already stated, is conclusive in favour of the latter theory.

The principle of interference furnishes the complete answer to the difficulty to which we alluded in our last lecture, and shows in what manner the *rectilinear propagation* of light is reconciled to the wave-theory. It had been objected, that if light consisted in the undulations of an elastic fluid, it should diverge in every direction from each new centre, and so bend round interposed obstacles, and obliterate all shadow. To this we reply, that light *does* diverge in every direction from each new centre,—that it *does* bend round interposed obstacles; but that *shadows* notwithstanding *exist*, because the several portions of this laterally diverging light *destroy one another* by interference, and *no effect* is produced

except by those parts of the wave which are in the right line joining the luminous origin and the eye.

To see this, let *abcd* (fig. 9) represent a portion of a spherical wave, and let *o* be the place of the eye, and *oa* the line drawn from it to the luminous centre. Commencing from the point *a*, let portions *ab*, *bc*, *cd*, &c. be taken, such that the differences of the distances of their extremities from the point *o* shall be the same for all, and just equal to *half a wave*. Now we may suppose all these portions of the grand wave to be so many centres of disturbance, and it is obvious that the *secondary* waves sent by each pair of consecutive portions to the eye will be in complete discordance; and, therefore, that they must wholly destroy one another if their intensities were equal. Now it is easy to see that this is the case with respect to the portions, as *fg*, *gh*, which are remote from the point *a*. For the magnitude of the waves sent by the several portions to any point depend—first, on the *magnitude* of these portions; and secondly, on the *angle* which the line drawn from them to that point makes with the front of the wave. With respect to the former, it is evident that *ab* is greater than *bc*, *bc* than *cd*, and so on; but that these differences go on continually diminishing, and that the magnitudes of the succeeding portions approach indefinitely to equality, as they recede from the point *a*. The same is true of the *obliquities*;—so that the portions of the wave, *fg*, *gh*, remote from the point *a*, destroy one another's effects, and the effect on the eye, or on a screen at *o*, will be entirely due to those parts of the grand wave which are in the neighbourhood of the line connecting that point with the luminous origin.

Of these parts *ab* produces, of course, the greatest effect, being both the largest and the least oblique. The effect of the neighbouring portions is, however, very sensible, and we shall have occasion in our next lecture to study some important phenomena to which they give rise. In the mean time

you cannot fail to perceive one remarkable consequence of this explanation—namely, that if the alternate portions *bc, de, &c.*, whose effects are *negative*, be stopped, the total effect will be *augmented*, and the light will be literally increased by intercepting a portion of the wave. On a future occasion we shall see that this startling conclusion is fully confirmed by experiment.

LECTURE III.

ON THE DIFFRACTION OF LIGHT.

It has been already shown to be a result of the wave-theory, that the intensity of the light which encounters an obstacle must diminish rapidly within the edge of the geometric shadow. It now remains to consider the other phenomena which arise under these circumstances; and it will be found that the same theory affords the most complete account, not only of their general characters, but even of their most minute details.

In order to understand the theory of *shadows*, it is necessary to investigate their laws in the simple case in which the magnitude of the luminous body is reduced to a point. The effects thus presented were first observed, and in some degree explained, by Grimaldi; and they have been since studied, as a separate branch of optical science, under the title of *diffraction* or *inflexion*.

Grimaldi found, that when a small opaque body was placed in the cone of light, admitted into a dark chamber through a very small aperture, its shadow was much larger than its geometric projection; so that the light suffered some deviation from the rectilinear course in passing by the edge. On observing these shadows more attentively, he found that they were bordered with three iris-coloured

fringes, which decreased in breadth and intensity in the order of their distances from the shadow, and which preserved the same distance from the edge throughout its entire extent, unless where the body terminated in a sharp angle. Similar fringes were observed, under favourable circumstances, *within* the shadows of narrow bodies. The phenomena of diffraction were subsequently examined by Hooke and by Newton; and, lastly, in the hands of Young and Fresnel, they have been forced to furnish evidence in favour of the wave-theory, which few who impartially examine it can, I think, continue to withstand.

Proceeding in the method which I have laid down to myself at the commencement, I propose, in the first place, to describe and illustrate the most important of these phenomena, and afterwards to examine them in their bearing upon the two theories.

The first of these phenomena to which I will call your attention, are the modifications which light undergoes in passing by the edge of an obstacle of any kind.

Let a beam of homogeneous light, entering a dark chamber, fall on a lens of short focal length, MN , (fig. 10,) by which it is brought to a focus at o , and thence diverges. Let an obstacle, rr' , be placed in the diverging beam, and let the shadow which it casts be received upon a sheet of white paper at q , or on a piece of roughened glass. We shall then observe the following phenomena :

1. The line orq , which is the boundary of the *geometric shadow*, is not the actual boundary of light and shade.
2. The space below this line, qs , is not absolutely dark, but is enlightened by a faint light, which extends to a sensible distance within the geometric shadow, and gradually fades away as it recedes from the edge of this shadow at q .
3. On the other side of the boundary of the geometric shadow, at qr , the paper is not uniformly illuminated by the diverging beam, but is observed to be covered with a series

of alternate bright and dark bands, which are parallel to the edge of the shadow. The distances of these fringes *inter se*, and from the edge of the shadow, vary with the position of the screen, and diminish indefinitely as the screen is brought near the obstacle. These fringes may be observed succeeding one another for many alternations; becoming, however, less marked as the distance from the edge of the geometric shadow increases, until at length they are wholly obliterated and lost. They preserve the same distance from the shadow in all parts, except only where the edge of the body forms a sharp angle.*

4. The fringes vary with the colour of the light; being broadest in *red* light, narrowest in *violet* light, and of intermediate magnitude in the light of mean refrangibility. Hence when *white*, or compound light, is employed, the fringes of different colours will not be accurately superposed; and there will therefore be a succession of *iris-coloured* fringes, the colours following the order which they have in the spectrum.

In this experiment we have made the incident light diverge from a very minute luminous origin. If the dimensions of the luminous origin had been considerable, it will be easily understood that each line in it, parallel to the edge of the obstacle, would give rise to a different system of fringes; and, as the dark bands of some of these systems must coincide with the bright bands of others, every trace of the phenomenon would be obliterated.

If we follow the course of the fringes from their origin, we shall observe that they are propagated in lines *sensibly curved*, whose concave side is turned towards the shadow. In order to obtain accurate measures of the dis-

* If this angle is *salient*, the fringes, instead of forming a similar angle, are observed to curve round the shadow. When the angle is *re-entrant*, they cross, and enter on the shadow at each side, without interfering with one another.

tances of the fringes from the edge of the shadow, at different distances from the obstacle, Fresnel found it convenient to view them directly with an eye-piece, instead of receiving them on a sheet of paper, or plate of roughened glass. This eye-piece was furnished with a micrometer, by means of which the distances of the fringes could be measured with the greatest nicety. It was ascertained in this manner that the curved path of each fringe was an *hyperbola*, whose summit coincided with the edge of the obstacle, and whose centre was the middle point of the line connecting that edge with the luminous origin.

If we consider these hyperbolas as coincident with their asymptots, (which may be done without sensible error, unless near the edge of the obstacle,) and if we then determine the angles which they make with one another, and with the edge of the geometric shadow, we shall find that these angles increase rapidly as the distance of the obstacle from the luminous point diminishes. When this distance is about 40 inches, the fringes are very close together; the fringes of the first and second order making an angle with one another of less than $2'$ in red light. At the distance of 4 inches, this angle is increased to more than $5'$; and at $\frac{1}{10}$ of an inch it exceeds $16'$. Thus the fringes dilate as the edge of the obstacle approaches the luminous origin.

The preceding experiments exhibit the effect of a *single* edge. When the light diverging from the luminous point is suffered to pass by *two near edges*, the phenomena will be varied in a very interesting manner.

Let a fine wire be placed in the beam of light diverging from a luminous point, and let its shadow be received on a screen or plate of roughened glass, as before. We then observe, outside the geometric shadow, a set of parallel bands, or fringes, analogous to those produced by the single edge in the former experiment. These are called *exterior fringes*.

But we observe further that the whole space of the geometric shadow itself is also occupied by parallel stripes, alternately bright and dark. These are called the *interior fringes*; and they are in general closer, and more finely marked than the exterior. When the breadth of the obstacle becomes at all considerable, the interior fringes disappear, and the phenomena fall under the class already examined.

The interior fringes are propagated, like the exterior, in hyperbolic curves; but their curvature is less considerable, and the deviation from a right-lined course scarcely perceptible within the limits at which they are commonly observed. They are also, like the exterior fringes, broader in red than in violet light, and of intermediate breadths in the light of intermediate refrangibility. Accordingly, in compound or white light, these fringes of different dimensions are superposed; and the bands are no longer alternately bright and black, but coloured with different tints, in the order of the colours of the spectrum.

When the obstacle is not terminated by parallel sides, the effect becomes more complicated. Thus two series of curved fringes are observed within the shadow of an object bounded by a salient angle, one on each side of the bisecting line. These fringes have been called the *crested fringes* of Grimaldi, by whom they were first observed.

It still remains to examine the effects which will be produced by two edges turned *inwards*, so as to form an *aperture* of any dimensions. For this purpose we are furnished with an instrument consisting of two metallic plates, one of which is fixed in the frame of the apparatus, while the other is moveable by means of a fine screw. The edges of these plates are right-lined and parallel, so that they form always a rectangular aperture; and, by means of the adjusting screw, the magnitude of this aperture may be varied at pleasure.

Now if a narrow rectangular aperture, thus formed, be

substituted for the wire in the last experiment, the resulting phenomena are very remarkable. It will be observed, in the first place, that the luminous beam has diverged considerably after passing the aperture, so that the space which it occupies on the screen, or roughened glass, is much wider than the geometric projection of the aperture. Secondly, it will be seen that the entire of this space is covered with parallel bands, or fringes, alternately bright and dark, distributed symmetrically on either side of the line passing through the luminous point and the centre of the aperture.

If we trace these fringes, from their origin at the aperture to any distance, we shall find that they are propagated in hyperbolas, like the former. The curvature of these hyperbolic branches, and their inclination to one another, depend on the breadth of the aperture, and on its distance from the luminous point. Fraunhofer, who observed this class of phenomena with great attention and care, found that the angular distances of the *successive* bands of any given colour from the central line formed an *arithmetical progression*, whose common difference was equal to its first term; and that, when different apertures were used, the distances of one and the *same* band from the central line were *inversely as the breadths of the apertures*. These fringes are broadest and most widely separated in red light; they are narrowest and closest in violet light, and of intermediate magnitude in the intermediate rays of the spectrum. In white light, therefore, they present the iris succession of colours observed in other cases.

When the aperture is formed by two straight edges *slightly inclined*, Newton observed that the fringes were not accurately parallel to the edges, but became broader as they approached; and that they finally crossed, and formed two hyperbolic branches, one of whose asymptots is perpendicular to the line bisecting the angle of the edges, while the others are parallel to the edges themselves.

It is scarcely necessary to observe that the phenomena of diffraction may be endlessly varied by varying the form of the diffracting edge. The preceding cases have been selected as the most elementary. They are abundantly sufficient, when pursued into numerical details, to test the truth of any theory which may be applied to this class of phenomena; and such a theory being once established, the laws of the more complex appearances are best sought for in its deductions. We shall now proceed, therefore, to consider the preceding phenomena in their relation to the two theories of light.

Newton conceived the rays of light to be *inflected* in passing by the edges of bodies, by the operation of the attractive and repulsive forces which the molecules of bodies were supposed to exert upon those of light before they arrived in actual contact. By the operation of such forces, you will remember, Newton was enabled to explain the laws of reflexion and refraction; and it was reasonable to suppose that the same forces played an important part in the phenomena now under consideration. Thus, the rays passing by the edges of a narrow opaque body, such as a hair or fine wire, are supposed to be turned aside by its repulsion; and, as this force decreases rapidly as the distance increases, it follows that the rays which pass at a distance from the body will be less deflected than those which pass close to it; see (fig. 11.) The *caustic* formed by the intersection of these deflected rays will be concave inwards; and as none of the rays pass within it, it will form the boundary of the *visible shadow*. Thus we see that this supposition explains in a satisfactory manner the *curvilinear* termination of the visible shadow, and its excess above the geometric one.

To account for the *fringes* which are parallel to the edge of this shadow, Newton appears to have supposed the attractive and repulsive forces to succeed one another for some alternations; and the molecules composing each ray, in

their passage by the body, to be bent to and fro by these forces, in a serpentine course, and to be finally thrown off at one or other of the points of contrary flexure. The intersection of the rays thus thrown off at different points of the same serpentine course will form a caustic or fringe; so that each succeeding fringe will be produced by the rays which pass at a given distance from the edge of the body. Finally, the separation of white light into its elements is explained by supposing that the rays which differ in refrangibility, differ also in *inflexibility*,—the body acting alike upon the less refrangible rays at a greater distance, and upon the more refrangible rays at a less distance.

It is needless to comment upon the vagueness of these explanations. Newton himself was dissatisfied with them, and the subject fell from his hands unfinished. Still, however, the mere guesses of such a mind as that of Newton must always possess a deep interest; and we are not to wonder that among his followers more weight should be attached to these conjectures than he himself ever assigned them. It seems necessary, therefore, to advert to some of the circumstances of these phenomena, which are not only unexplained by this theory, but which seem moreover at utter variance with it.

If the phenomena of inflexion be the effects of attractive and repulsive forces emanating from the interposed body,—and if these forces are the same, or even analogous to those to which the reflexion and refraction of light are ascribed in the theory of emission, it will follow that they must exist in different bodies in *very different degrees*; so that the amount of bending of the rays, and therefore the breadth of the diffracted fringes should vary with the *mass*, the *nature*, and the *form* of the inflecting body. Now it is clearly ascertained, on the contrary, that *all bodies*, whatever be their nature or the form of their edge, produce, under the same circumstances, fringes *identically the same*; and, in fact, the partial interruption of light caused by the interposition of an ob-

stacle of any kind, appears to be the only condition essential to the phenomenon. Gravesende seems to have first observed that the *nature* or *density* of the body had no effect upon the magnitude of the diffracted fringes; and the fact has since been confirmed in the fullest manner by almost every inquirer in this branch of experimental science. One of the ablest of the supporters of the theory of emission has admitted that the inflecting forces, if such exist, must be independent of the chemical nature of the inflecting body, and altogether different from those to which, in the same theory, the phenomena of reflexion and refraction are ascribed.

To ascertain whether the *form of the edge* had any effect upon the fringes, Fresnel took two plates of steel, the edge of each of which was rounded in one half of its length, and sharp in the remaining half,—and placed the rounded portion of each edge opposite the angular part of the other. If then the position of the fringes depended on the form of the *surface*, the effect would thus be doubled, and the fringes appear broken in the midst. They were found, on the contrary, to be perfectly straight throughout their entire length.

Again, the inflecting forces (though they must be supposed to vary in intensity with the form and mass of the body, and with the distance of the luminous molecule from the edge) cannot be conceived to depend in any way upon the *distance previously traversed* by the molecule before it arrives in the neighbourhood of that edge; so that the magnitude and position of the fringes, in this hypothesis, cannot vary in any way with the distance of the inflecting edge from the luminous point. But this conclusion is the reverse of fact. The fringes *dilate*, and their mutual inclination is increased, as the obstacle approaches the luminous origin.

The phenomena of diffraction, therefore, do not arise

from the operation of attractive and repulsive forces, exerted by the molecules of body upon those of light. The same objections apply to the hypothesis of Mairan and Du Tour, which ascribes these effects to the *refraction* of small atmospheres encompassing the body, and of a different refractive power from the surrounding medium. For, if such an atmosphere be retained by the attraction of the body which it encompasses, (and this seems to be the only intelligible mode of accounting for its presence,) its density and its form must vary with those of the body itself; and, consequently, its effects upon the rays of light must vary also. We are forced, then, to conclude, that the phenomena of diffraction are inexplicable in the system of emission.

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LECTURE IV.

ON THE DIFFRACTION OF LIGHT.

GENTLEMEN,—In my last lecture I called your attention to what may be termed the *elementary* phenomena of light, in its passage by the edges of bodies. I stated the principal laws of these phenomena as discovered by experiment, and showed in what manner they were verified; and, finally, I gave a brief sketch of the unsuccessful attempt to explain these laws by the operation of attractive and repulsive forces, in the theory of emission. I now propose to show in what manner, and with what success, the principles of the wave-theory have been applied to the same problem.

This important step in physical optics was made by Dr. Thomas Young, and all the complicated phenomena of diffraction are now reduced to the simple *principle of interference*. The *exterior* fringes, formed without the shadows of bodies, were ascribed by Young to the interference of two portions of light, one of which passed by the body, and was more or less deviated, while the other was obliquely reflected from its edge. The fringes formed by narrow apertures were, in like manner, supposed to arise from the interference of the two pencils reflected from the opposite edges; while the *interior* fringes, within the shadows of narrow bodies, were accounted for by the interference of the pencils which passed on either side of the body, and were

bent into the shadow. The observed facts closely correspond with the calculated results of this theory; and in the case last mentioned, Young proved that the phenomenon admitted no other explanation. Placing a small opaque screen on either side of the diffracting body, so as to intercept the portion of light which passed by one of its edges, the whole system of bands immediately disappeared, although the light passing by the other edge was unmodified.

The *general* laws of the fringes,—the dependence of their magnitude upon the length of a wave, and upon the distances of the luminous origin and of the screen,—are fully explained on these hypotheses. It is easy to infer from them that as the position of the screen is varied, the successive points of the same fringe are not in a right line, but constitute an hyperbola; and that when the distance of the luminous origin is lessened, the inclination of these hyperbolic branches (considered as coincident with their asymptots) augments, and the fringes dilate.

Fresnel at first adopted and developed the hypotheses of Young; but he soon found reason to be dissatisfied with them. If the exterior fringes arose from the interference of the direct light with that obliquely reflected from the edge of the obstacle, it would follow that the intensity of the light in them should depend on the extent and curvature of the edge. It was found, on the contrary, that the fringes were wholly independent of the form of the diffracting edge; the fringes formed by the back and by the edge of a razor, for example, being precisely alike in every respect. In the other cases of diffraction also, there were many phenomena which proved that the rays grazing the edge of the body were not the only rays concerned in the production of the fringes; but that the light which passed by these edges at sensible distances was also deviated, and concurred in their formation.

Fresnel was thus forced to seek a broader foundation

for his theory, and the result of his investigations is given in the able memoir which was crowned by the French Academy in 1819. In this theory the phenomena of diffraction are ascribed to the interference of the partial, or secondary waves, which are separated from the grand wave by the interposition of the obstacle. In applying this principle, he supposes the surface of the wave, when it reaches the obstacle, to be subdivided into an indefinite number of equal portions. Each of these portions may, by the principle of Huygens, be considered as the centre of a system of partial waves; and the mathematical laws of interference enable us to compute the resultant of all these systems at any given point. This resultant vibration, Fresnel has shown, is in general expressed by means of two integrals, which are to be taken within limits determined by the particular nature of the problem. Its square is the measure of the intensity of the light; and it is found that its value has several maxima and minima, which correspond to the intensities of the light in the bright and dark bands.

The problem of diffraction was thus completely solved; and its laws derived from the two principles to which the laws of reflexion and refraction are themselves referred,—the *principle of interference* and the *principle of Huygens*. It only remained to apply the solution to the principal cases, and to compare the results with those of observation. The cases of diffraction selected by Fresnel are those whose laws were developed in our last lecture; viz. the phenomena produced—1. by a single straight edge; 2. by an aperture terminated by parallel straight edges; and 3. by a narrow opaque body of the same form. The agreement of observation and theory is so complete, that the computed places of the several bands seldom differ from those observed by more than the 100th part of a millimetre.

The general circumstances of these phenomena may be deduced by very simple considerations from the principles already laid down; although the complete development

of these principles demands the aid of a complicated analysis.

Thus, in the case of the fringes produced by a single edge, let o (fig. 12) be the luminous origin, MAN a diverging wave, and R any point at which the illumination is sought. From this point, as centre, let a circle be described, touching the circle MAN in a , and let the lines $Rb'b$, $Rc'c$, &c. be drawn in such a manner that the intercepts bb' , cc' , dd' , &c. are equal respectively to one, two, three, &c. semiundulations. The effect produced at the point R is then (by the principle of Huygens) the sum of the effects produced by each of the portions ab , bc , cd , &c. separately. But, the distances of these consecutive portions from the point R differing by half a wave, their effects will be opposed at that point; so that, if m denote the intensity of the light sent from the portion ab , m' that from bc , &c. (the light sent from the indefinite wave, AM or AN , being taken as unity,) the actual light which reaches the point R will be $1, 1+m, 1+m-m', 1+m-m'+m'',$ &c. according as the obstacle is placed at the points a, b, c, d , &c.; and the intensity of the light when the obstacle is altogether withdrawn is

$$1 + m - m' + m'' - m''' + \&c. = 2.$$

Now, as the terms of this series go on continually decreasing, and are affected alternately with opposite signs, it is manifest that if we stop at any term, the sign of the remainder will be the same as that of its first term, and therefore alternately positive and negative. Accordingly, the intensities, $1+m$, $1+m-m'$, $1+m-m'+m''$, &c. are alternately greater and less than 2; and the intensity of the light sent to the point R is alternately greater and less than when no obstacle is interposed.

It will be easily understood, from this general explanation, in what manner the magnitude of the fringes depends on the length of a wave, on the distance of the luminous origin from the obstacle which produces the fringes, and on the distance

of the screen on which they are received. We see that they must be broadest in *red* light, in which the length of an undulation is *greatest*, and narrowest in *violet* light, in which the length of an undulation is *least*. When white or compound light, therefore, is employed, the diffracted bands of different colours will occupy different positions; so that there will be a succession of *iris-coloured* bands having the violet or blue inside, and the red without. After a few successions these bands wholly disappear, owing to the superposition of bands of different colours.

It is easy to compute the relative places of the same fringe, for different positions of the luminous point, and of the screen.

Let *P* (fig. 13) be the edge of the obstacle, *PA* a portion of the wave diverging from *o*, which has just reached that edge; and let *QR* be the screen, and *R* the place of a fringe of any given order. Then, in order that this point should belong to the same fringe for every distance of the luminous origin and of the screen, it is only necessary that the interval of retardation (*RP*—*RA*) of the central and marginal parts of the wave, should be constant. For in this case the whole wave, *AP*, may be divided into a given number of parts, such that the difference of the distances of the successive points of division from the point *R* shall be constant; and therefore the effective wave consists of the same number of elementary portions in the same relative state as to interference.

Now, denoting *OP* by *a*, *PQ* by *b*, and *QR* by *x*, we have

$$RP = \sqrt{b^2 + x^2} = b + \frac{x^2}{2b}, \quad q.p.$$

since *x* is very small in comparison with *b*. Similarly

$$RO = \sqrt{(a+b)^2 + x^2} = a+b + \frac{x^2}{2(a+b)};$$

so that $RA = b + \frac{x^2}{2(a+b)}$, and $RP - RA = \frac{1}{2} x^2 \left(\frac{1}{b} - \frac{1}{a+b} \right)$.

But, by the conditions of the question, this difference is a constant quantity; and, denoting this constant by δ , we have

$$\frac{1}{2} x^2 \left(\frac{1}{b} - \frac{1}{a+b} \right) = \delta, \quad x = \sqrt{\frac{2\delta(a+b)b}{a}}.$$

When b varies, a remaining unaltered;—i. e. when the position of the screen is varied,—the value of x is the ordinate of an hyperbola whose abscissa is b ; so that the successive points of the same fringe belong to an hyperbola, whose summit is the edge of the obstacle.

The interior fringes formed in the shadow of a narrow opaque body arise, it has been said, from the interference of the two portions of the wave which pass by the edges on either side. Let pr' (fig. 14) be the section of the opaque body, pc and $p'c'$ the two portions of the diverging wave which has just reached its edges, and let r be any point of the shadow. Then, if these portions be divided in the points a, b, c , &c. a', b', c' , &c. in such a manner that the difference of the distances of any two consecutive points from the point r is equal to half an undulation, the elementary wave sent from each portion will be in complete discordance with those sent from the two adjoining portions; so that, if the several portions be equal, they will neutralize one another's effects at the point r , with the exception of the extreme portions, pa , $p'a'$, the halves of which adjoining the edges remain uncompensated.

Now the arcs, pa , ab , bc , &c. are very nearly equal, when the lines drawn from their extremities to the point r are sufficiently inclined to the normal,—or, in other words, when this point is sufficiently removed from the edge of the geometric shadow. In this case, then, the only efficacious parts of the wave are the halves of the extreme portions, pa and $p'a'$; and the intensity of the light at the point r will be determined by the difference of their distances from that point, or (which comes to the same thing) by the difference of the lengths of the lines connecting it with the edges of the

obstacle. The phenomena of interference are therefore the same as in the case of light emanating from two near origins, already considered; and we may transfer to the present case the conclusions at which we arrived in a former lecture. Accordingly, if c denote the breadth of the obstacle, and b its distance from the screen, the distance (x) of any band from the centre of the shadow is

$$x = \frac{nb\lambda}{2c}.$$

The position of the fringes formed by a narrow rectangular aperture will be determined by a similar formula.

Let PP' (fig. 15) be the section of the aperture, PAP' the portion of the wave which has just reached it, diverging from the luminous origin at o ; and let QQ' be the projection of the aperture on the screen. Then, if we take the point R on this screen in such a manner that the difference of its distances from the edges of the aperture, $RP' - RP$, shall be equal to a whole number of semiundulations, that point will be the centre of a *dark* or *bright* band, according as the assumed number is *even* or *odd*. For, in the former case, the wave PAP' may be divided into an even number of parts, such that the distances of every two consecutive points of division from the point R differ by half an undulation. The waves sent by every two consecutive portions to the point R will therefore be in complete discordance, and the total effect at that point will be null. On the other hand, when the difference $RP' - RP$ is equal to an odd number of semiundulations, the number of opposing portions of the wave will be odd, and as the alternate portions compensate each other's effects at the point R , there will remain one portion producing there its full effect.

Since the successive bands are formed at the points for which $RP' - RP = n\lambda$, it is obvious that their distances, RB , from the centre of the projection of the aperture will be

given by the same formula as in the case last considered; c being now the breadth of the aperture. There will be this remarkable difference however, that the dark bands correspond to the even values of n , and the bright bands to the odd values, which is the reverse of what takes place in the bands formed within the shadow of an opaque obstacle. We see then, 1st, that the distances of the successive fringes of any one colour form an arithmetical progression whose common difference is equal to its first term: 2dly, that they vary directly as the distance of the screen, and inversely as the breadth of the aperture; and, 3dly, that they are proportional to the length of a wave; and therefore greatest for the extreme red rays, least for the extreme violet, and of intermediate magnitude for the rays of intermediate refrangibility.

We have here supposed the screen to be so remote, that the bands are entirely *without* the projection of the aperture. This will obviously be the case when $qr' - qp$ is less than half a wave. When the distance of the screen becomes so small that $qr' - qp$ exceeds this limit, fringes will be visible also *within* the projection of the aperture. In this case the portions into which the wave is divided are sensibly different in magnitude, as well as obliquity; the reasoning above employed is therefore no longer applicable, and the points of maximum and minimum brightness can only be obtained by a complete calculation of the intensity of the light.

The phenomena of diffraction which we have been hitherto considering are of the simplest class;—but as such phenomena arise in every instance in which light is in part intercepted, it is obvious that they admit of endless modifications, varying with the form of the interposed body. Some of these are too remarkable to pass unnoticed.

Among the most striking of these phenomena are those produced by light diverging from a luminous origin, and transmitted through a small *circular aperture*;—as, for example, that formed by a pin in a sheet of lead. When the

transmitted light is viewed through a lens, the image of the aperture appears as a brilliant spot, surrounded by coloured rings of great vividness; and these vary in the most beautiful manner, as the distance of the aperture from the luminous origin, or from the eye, is altered. When the distance of the eye from the aperture is considerable, the central spot is white; and the coloured rings follow the order observed in thin plates. As the eye approaches the aperture, the central white spot contracts to a point, and then vanishes. The rings then close in on it in order, and the centre assumes in succession the most vivid and beautiful hues, altogether similar to those of the reflected rings of thin plates.

This remarkable coincidence has been shown to be an exact result of theory. It has been demonstrated that the intensity of the light of any simple colour, at the central spot,—and the compound tint in the case of white light,—will be the same as that reflected from a plate of air, whose thickness bears a certain simple relation to the radius of the aperture, and its distances from the luminous origin and from the eye. With homogeneous light, therefore, the illumination of the central spot vanishes periodically, as the distance of the eye from the aperture is varied.

These conclusions are easily established. Let o (fig. 16) be the luminous point, and oab the line drawn from it through the centre of the aperture PF' ; then the interval of retardation (δ) of the ray which reaches any point B on this line, coming from the edge of the aperture, is $OP + PB - OB$. Let $OA = a$, $AB = b$, and $AP = r$; then, since r is very small in comparison with a and b , it is easy to see that

$$OP = a + \frac{r^2}{2a}, \quad BP = b + \frac{r^2}{2b}, \quad \delta = \frac{1}{2} r^2 \left(\frac{1}{a} + \frac{1}{b} \right).$$

Now when this interval is equal to a whole number (n) of semiundulations, the aperture may be divided by concentric

circles, such that the rays which reach the point *B* coming from any two successive circumferences, shall differ by the interval of half a wave. It is obvious from the preceding formula that the squares of the radii, and therefore the superficies of the successive circles thus formed, will be as the numbers 1, 2, 3, 4, &c. of the natural series; so that the annuli comprised between every two succeeding circumferences are equal. But the elementary waves proceeding from each annulus are in complete discordance with those from the adjacent. The successive annuli will therefore destroy one another's effects, and the total intensity of the light at the point *B* will be null, or equal to that of the last, according as the number of annuli (the central circle included) is even or odd. Hence, for a given aperture, there will be a succession of points on the axis, at which the intensity of the light is alternately *nothing* and a *maximum*; and it is obvious from the preceding that the distances of these points will be the values of *b* given by the formula

$$r^2 \left(\frac{1}{a} + \frac{1}{b} \right) = n\lambda;$$

in which the points of complete darkness correspond to the even values of *n*, and those of maximum brightness to the odd values.

Such is the case with homogeneous light. As the points of maximum and minimum intensity are different for the rays of different colours, there will be no point of complete darkness in compound light, but a succession of points, at which the centre of the aperture is richly coloured.

But the theory of Fresnel is not only in exact accordance with facts already known; it has also led to many new and unexpected conclusions, and predicted consequences which have been afterwards verified on trial. One of the most remarkable of these is the phenomenon of diffraction by an opaque circular disc. M. Poisson applied

Fresnel's integrals to this case; and he was led to the startling result, that the illumination of the centre of the shadow was precisely the same as if the disc had been altogether removed. The principles already laid down will easily enable you to satisfy yourselves of the theoretical truth of this conclusion. M. Arago was the first to show that it was in accordance with fact, and his experiment may be repeated without difficulty.

We have seen that when light diverging from a luminous point passes by the edges of a fine hair or wire, a succession of coloured bands will be formed parallel to the edge of the shadow; and the distances of these bands from the shadow and from one another will be greater, the less the diameter of the wire. If many such wires be exposed to the diverging beam, and if, instead of being parallel, they are crossed and interlaced in every possible direction, it is easy to conceive that the coloured bands will be disposed in concentric circles, whose centre is the luminous point. These circles resemble the halos visible round the sun and moon in hazy weather. Their diameters depend on the thickness of the wires or fibres, and vary in the inverse ratio of that thickness.

These phenomena were observed by Dr. Young, and were applied by him in a very ingenious manner to the comparison of the diameters of fibres and small particles of any kind. A plate of metal is perforated with a small round hole, about the $\frac{1}{30}$ th of an inch in diameter, around which, at the distance of about $\frac{1}{2}$ or $\frac{2}{3}$ of an inch, is a circle of much smaller holes. The flame of a lamp is then placed immediately behind the aperture, and the luminous point viewed through the substance to be examined. A ring or halo will be seen surrounding the aperture; and by moving the substance backwards and forwards on a graduated ruler, this ring may be brought to coincide with the circle of small holes pierced in the plate. The distance from the aperture

is then read off on the ruler, and varies obviously in the inverse ratio of the angular diameter of the spectrum. But the diameters of the particles vary also in the same inverse ratio,—so that the distance on the ruler at once becomes a measure of these diameters. In this manner Dr. Young compared the diameters of a great number of very minute substances,—such as the fibres of the finest wools, the globules of the blood, &c. The instrument itself he called the *Eriometer*.

In the case last mentioned, we have supposed the intervals of the fibres, or fine wires, to be much greater than their thickness; in which case the phenomenon depends mainly on the diameter of the opaque fibre. When the intervening apertures are very small, the effect is influenced by *their* magnitude, and assumes a different character. Thus, if a *grating* be formed, by stretching a fine wire between two fine screws of equal thread, and if this grating be held in the beam diverging from a luminous point, we shall observe, on either side of the direct image, a series of spectral images richly coloured with all the prismatic tints; the spectra increasing in breadth, and diminishing in intensity, as they recede from the centre.

These phenomena are seen to most advantage by means of a telescope adjusted to the luminous origin. The grating being held before the object-lens of the telescope, the spectra are formed at its focus, and are there viewed, with all the advantages of distinctness and amplification, by means of the eye-lens. Fraunhofer, who observed these phenomena with much attention and care, traced no fewer than thirteen spectra on either side of the central image; the first pair being separated from the central image, and from the second pair, by intervals absolutely black. By a very accurate mode of measurement the same philosopher ascertained that the deviations of any one colour from the central image, in the successive spectra, formed an arithmetical progression;

and that the absolute amount of these deviations depended on the intervals of the axes of the wires, varying inversely as that interval.

These results flow readily from the principle of interference;—the *first* pair of spectra on either side of the central image being produced by the interference of those rays whose paths differ by *one* undulation, the *second* pair by those whose paths differ by *two* undulations, and so on. Let the light proceeding from a very remote origin fall on the grating, whose opaque parts are represented by $ab, a'b', a''b'',$ &c. (fig. 17); and let q be the place of the eye. Then, if we take a portion of the grating, $a'a''$, composed of one opaque and one transparent interval, in such a manner that the difference of the distances of its extremities from the point q ($qa'' - qa'$) shall be equal to the length of a wave, it is manifest that the corresponding portion of the incident wave, $a'a''$, may be divided into two parts very nearly equal, the waves sent from which to the point q shall be in complete discordance. Without the grating, therefore, the effect of that portion of the incident wave would be null at the point q , and no light from it would reach the eye. The effect of the grating, however, is to intercept the whole or part of one of the two interfering portions, and thus to give effect to the other; and this effect is greatest when the opaque and transparent parts of the grating are equal. A bright band will therefore be visible in the direction qa'' . It is obvious that the same thing will happen for all the similar divisions of the grating, the distances of whose extremities from the point q differ by two, three, or any whole number of undulations; and that thus there will be a succession of bright bands visible at different angular distances from the direct ray pq .

These angles are easily computed. Let $a'k$ be the arch of a circle described with the centre q ; then $a''k = a'a'' \cos a'a''k = a'a'' \sin pqa''$. But the interval of retardation, $a''k$,

is equal to the length of a wave; so that, if the angle $\text{PQ}a''$ be denoted by θ , and the interval composed of an opaque and transparent part of the grating, $a'a''$, by ϵ , we have

$$\sin \theta = \frac{\lambda}{\epsilon}.$$

This is the angular distance of the *first* bright band from the central one; and it is obvious that the corresponding angle, θ_n , for the band of the n^{th} order is given by the formula

$$\sin \theta_n = \frac{n\lambda}{\epsilon}.$$

It is a remarkable circumstance of the phenomenon whose laws we have been tracing, that when the experiment is performed with the requisite care, the several species of homogeneous light are so pure and unmixed in the spectra, that the *fixed lines* may be discerned. These lines, then, are wholly independent of refraction, and exist in the parts of the solar beam before they are separated by the prism. The phenomenon, when thus exhibited, is however distinguished by a remarkable peculiarity. The distances of the fixed lines in the diffracted spectrum are always *proportional*, whatever be the diffracting substance; while the ratios of the intervals of the fixed lines, (or of the breadths of the coloured spaces,) in the spectra formed by refraction, vary with the dispersive power of the prisms.* The diffracted spectrum, therefore, becomes an important optical standard, to which all others may be referred.

The formula above given suggests a very simple method of determining the length of the wave corresponding to any given ray of the spectrum. The value of ϵ , or the interval of the axes of the wires, may be ascertained with the greatest ease and precision; and we have therefore only to measure

* This fact alone would appear to be decisive against the Newtonian theory of inflexion, in which inflexion and refraction are referred to the same cause.

the angular deviation (θ_n) of the ray of any simple colour from the axis, in order to deduce the value of λ . Fraunhofer computed in this manner the lengths of the waves corresponding to the seven principal fixed lines of the spectrum; and the resulting values are perhaps the most exact optical constants we possess.

The beautiful appearances which you have witnessed may be varied in many ways by changing the form of the grating. Thus, if two gratings, composed of parallel and equidistant wires, be crossed at right angles, so as to form a grating with square apertures, the phenomenon displayed is one of remarkable splendour.

The optical phenomena of gratings are interesting in many points of view. The appearance of lateral spectra, produced by simply *intercepting part of the light*, proves that the light actually diverges in all directions from the front of the grand wave; and that it is to the interference of the two portions, of which one is intercepted by the grating, that we are to ascribe its want of sensible effect under ordinary circumstances.

Gratings producing these effects may be formed in several ways,—as, for example, by tracing a number of parallel lines on glass with a fine diamond point. Fraunhofer succeeded by such means in forming ruled surfaces in which the striæ were actually invisible under the most powerful microscopes, the interval of the grooves being only the $\frac{1}{30000}$ of an inch. Analogous phenomena may be produced by *reflexion*. If a great number of parallel lines be engraved at very small and equal intervals upon a polished surface, the light reflected from the intervals of the grooves thus formed will interfere in a manner precisely analogous to that admitted through the apertures of the gratings; and will by their interference produce the most brilliant spectra. In some of the grooved metallic surfaces constructed by Mr. Barton the lines are so close that 10,000 of them would

occupy only the space of one inch in breadth. With surfaces so minutely divided, the spectra produced are as perfect as those formed by the finest prisms; and the colours which they display by candle light are scarcely inferior to those of the diamond.

Similar appearances may be observed to a certain extent on metallic surfaces which have been polished with a coarse powder, the powder leaving minute striæ which produce the effects we have been describing. They may also be very simply produced by passing the finger over the surface of a piece of glass moistened by the breath. The striæ thus formed in the coating of vapour display very brilliant colours, which vary with the position of the eye.

The beautiful colours of *mother of pearl* are natural instances of the same phenomena. This substance is composed of a vast number of very thin layers, which are gradually and successively deposited within the shell of the oyster, each layer taking the form of the preceding. When it is wrought, therefore, the natural joints are cut through in a great number of sinuous lines; and the resulting surface, however highly polished, is covered by an immense number of undulating ridges, formed by the edges of the layers. These striæ may be observed by the aid of a powerful microscope, although they are sometimes so close that 5000 of them occupy an inch. That they are the causes of the brilliant colours displayed by this substance has been placed beyond doubt by an experiment of Sir David Brewster. This experiment consisted simply in taking the impression of the surface of the pearl on wax, or any other substance fitted to receive it; and it was found that the impressed surface displayed all the colours of the original body. In fact the colours of striated surfaces indicate their structure, perhaps more unerringly than any other means. Sir David Brewster has made a beautiful use of their laws in investigating the curious and compli-

cated structure of the crystalline lens in fishes and other animals.

There remains another class of phenomena produced by diffraction, to which I wish to call your attention before I conclude. We have already seen the effects produced when light diverging from a luminous point or line is transmitted through a narrow aperture, and received on a screen. But if we vary the experiment, by placing a lens of considerable focal length (as the object-glass of a telescope) immediately behind the aperture, and receive the image on a screen at the focus conjugate to the luminous origin, the appearances displayed are altered in a remarkable manner, and differ more widely from those produced in the former case, as the aperture is greater. In fact the phenomena of diffraction are thus produced with apertures of considerable dimensions, and were observed by Sir William Herschel with the undiminished object-specula of his great telescopes. They are rendered more distinct, however, when the aperture of the telescope is limited by a diaphragm of moderate size. When a star is viewed through a telescope of high power, with its object-lens thus limited, its image is encompassed with a system of diffracted rings slightly coloured, succeeding one another at equal intervals;—the diameters of the rings varying inversely as those of the apertures. The phenomena vary in a very curious manner, when the form of the aperture is changed. Thus, when a triangular diaphragm is substituted for the circular one, the disc of the star appears surrounded by a black ring, from which diverge six rays at equal intervals. These phenomena have been examined in detail by Sir John Herschel and M. Arago. Their mathematical explanation has been given by Mr. Airy, in his valuable tract on the undulatory theory; and the deductions of theory are found to be in complete accordance with the observed facts.

LECTURE V.

COLOURS OF THIN PLATES.

GENTLEMEN,—In the two preceding lectures I invited your attention to the effects which take place in the passage of light by the edges of an obstacle. In all such cases, it was shown, a series of alternate bright and dark bands is formed parallel to the edge; the bright bands being of the colour of the light when simple light is used, but in white light assuming all the varied tints of the spectrum. All these appearances, we found, were but cases of a more general property of light, known under the name of *interference*;—and we now proceed to consider a different class of phenomena, which are reducible to the same fertile principle.

The *colours of thin plates* were first noticed by Boyle and Hooke. They are displayed in every instance in which transparent bodies are reduced to films of great tenuity. Boyle succeeded in blowing glass so thin as to exhibit the phenomena: they are more readily developed in mica, and some other transparent minerals, which possess a lamellar structure; but the most familiar instance of their exhibition is in the froth of liquids,—the fluid envelope of the bubbles which compose it being in general of extreme thinness.

These colours vary with the thickness of the film, and disappear altogether when it passes certain limits. When

the film exceeds a certain thickness, all the colours are equally reflected, and the reflected light is therefore *white*. On the contrary, when the thickness falls below a certain limit, no light whatever reaches the eye, and the surface of the film appears absolutely *black*. These facts may be observed in the common soap-bubble, when properly defended from the disturbing influence of currents of air. If the mouth of a wine-glass be suddenly dipped in water, which has been rendered somewhat viscid by the mixture of soap, the aqueous film which remains in contact with it after emersion will display the whole succession of these phenomena. When held in a vertical plane, it will at first appear uniformly white over its entire surface; but, as it grows thinner by the descent of the fluid particles, colours begin to be exhibited at the top, where it is thinnest. These colours arrange themselves in horizontal bands, and become more and more brilliant as the thickness diminishes;—until finally, when the thickness is reduced to a certain limit, the upper part of the film becomes completely black. When the bubble has arrived at this stage of tenuity, cohesion is no longer able to resist the other forces which are acting on its particles, and it bursts.

Similar phenomena may be observed when a drop of oil is let fall on water. As it spreads rapidly over the surface, it is soon reduced to a very thin film, which displays the spectral colours.

Every one has noticed the fact that steel and other metals, when polished, acquire various shades of colour by exposure to the air. These colours are produced by a thin coating of metallic oxide which is gradually formed on the surface. The formation of this oxide is greatly accelerated by an augmentation of temperature; and the colour thus formed is so invariably connected with the thickness of the film, and this latter with the degree of heat, that artists are in the habit of measuring the temperature by the colour de-

veloped. Thus steel, in the process of tempering, is said to have received a *yellow heat*, a *blue heat*, &c.

The same appearances are displayed in a still more striking manner by air itself, or even by a vacuum. If two plates of glass be pressed together by the fingers, we shall observe, round the point of nearest approach, a succession of coloured bands of great brilliancy, which dilate as the pressure is increased, and the enclosed plate reduced in thickness.

But in order to observe these phenomena, in such a manner as to be enabled to trace their laws, we must follow Newton. Newton's experiment consisted simply in laying a convex lens of glass upon a plane surface of the same material. The thickness of the plate of air increases as the square of the distance from the point of contact, and is, therefore, the same at all equal distances from that point. Hence, as the reflected colour depends on the thickness, the bands of the same colour will be arranged in concentric circles of which that point is the centre. The same succession of colours is produced when any other transparent fluid is enclosed between the glasses. The colours, however, are more vivid, the more the refractive power of the plate differs from that of the substances within which it is enclosed.

When we look attentively at these rings, the light being reflected always at the same angle, we observe that the central one is not a mere annulus, but a complete circle of nearly uniform colour. If then we diminish the thickness of the plate of air, by pressing the two glasses more closely together, this central circle is observed to dilate, and a new circle of a different colour to spring up in its centre. This will dilate in turn, driving the former before it, and another circle appear within it;—until at length a *black spot* shows itself in the centre of the system, after which no further diminution of thickness will alter the succession. When the black spot makes its appearance, we have obtained a plate of

air so thin as no longer to reflect any colours, and the phenomenon is then complete. Newton traced seven coloured rings round this spot, the colours of which are said to be of the *first, second, third, &c. order*, according to the order of the ring to which they belong. Thus, the red of the third order is the red which is found in the third ring from the central black, &c. The whole succession of colours is called *Newton's scale*.

The principal laws of these phenomena are included in the following propositions :

1. In homogeneous light, the rings are alternately *bright* and *black*;—the thicknesses corresponding to the bright rings of succeeding orders being as the *odd* numbers of the natural series, and those corresponding to the black rings as the intermediate *even* numbers.

2. The thickness corresponding to the ring of any given order varies with the *colour* of the light,—being greatest in red light, least in violet, and of intermediate magnitude in light of intermediate refrangibility. In white or compound light, therefore, each ring will be composed of the rings of different colours, succeeding one another in the order of their refrangibility.

3. The thickness corresponding to the ring of any given order varies with the *obliquity*, being very nearly proportional to the secant of the angle of incidence.

4. The thickness corresponding to the ring of any given order varies with the *substance* of the reflecting plate, and in the inverse ratio of its refractive index.

In order to establish the first of these laws, it is necessary to employ homogeneous light. This may be obtained by means of the prism; or we may adopt the method suggested by Mr. Talbot, and illuminate the glasses with a spirit lamp having a salted wick. The light of such a lamp being a yellow of almost perfect homogeneity, the

rings will be alternately *black* and *yellow*; and their number is so great as to baffle any attempt to determine it.

The law of the thicknesses corresponding to the successive rings is easily established. Let o (fig. 18) be the point of contact of the plane and spherical surface, and ad' , bb' , cc' , &c. the diameters of the successive rings formed round that point as a centre. It is evident that the thicknesses of the plate of air at the points where these rings are formed, aa , $b\beta$, $c\gamma$, &c. are as the squares of the distances, oa , ob , oc , &c., or as the squares of the diameters of the rings. To determine the law of the thicknesses, therefore, we have only to measure these diameters. This was done by Newton with great accuracy, and it was found that their squares were in *arithmetical progression*; consequently, the thicknesses at which the successive rings were formed increased in a similar progression.

But Newton did not stop here. He ascertained further the *absolute thickness* of the plate of air at which each ring was formed, and that in the most exact manner. It is manifest that if the thickness of the plate be determined for any one of the rings, that corresponding to the others will be given by the law just stated. Newton, accordingly, proceeded to ascertain this thickness for the dark ring of the *fifth* order. This was done by measuring its diameter accurately, and determining the radius of the spherical surface from the focal length of the lens and its refractive index. These two elements being obtained, the thickness is immediately deduced; for it is equal to the square of the radius of the ring divided by the diameter of the spherical surface. The value thus deduced being suitably corrected, it was found that the thickness of the plate of air was the $\frac{1}{17800}$ of an inch, at the dark ring of the fifth order; and this thickness being decuple of that corresponding to the first bright ring, it followed that the thickness of the plate of air, where the *first bright ring* was formed, was the $\frac{1}{178000}$ of an inch. Thus

the *bright rings* of the successive orders are formed at the thicknesses

$$\frac{1}{178000}, \quad \frac{3}{178000}, \quad \frac{5}{178000}, \quad \frac{7}{178000}, \quad \&c.$$

and the intermediate *dark rings* at the thicknesses

$$\frac{2}{178000}, \quad \frac{4}{178000}, \quad \frac{6}{178000}, \quad \frac{8}{178000}, \quad \&c.$$

These determinations belong to the most luminous rays of the spectrum, or those at the confines of the orange and yellow. The variation of the diameters of the rings (or of the thickness of the plate of air at which they are exhibited,) with the *colour* of the light, may be observed by illuminating the glasses with different portions of the spectrum in succession,—or, yet more simply, by looking at the rings through coloured glasses;—and it will be found that the magnitude of the rings is greater, the less the refrangibility of the light. This being understood, it is easy to comprehend the cause of the succession of colours in each ring, when white or compound light is used. For the rings, in this case, are the aggregate of the rings of different colours; and these being of different magnitudes, the compound ring will be variously coloured, the more refrangible rays occupying the interior, and the less refrangible the exterior parts of the ring. It is easy to see also, that, in this case, all phenomena of colour must disappear after a few successions, the rings of different colours, belonging to different orders, being at length superposed.

The variation of the rings with the *obliquity* of the incident light may be observed by depressing the eye. The rings are then seen to dilate rapidly with the obliquity of the reflected pencil; the thickness of the plate of air at which they are exhibited being nearly as the secant of the angle of incidence or reflexion.

The fourth and last law, which expresses the dependence of the thickness, at which any ring is formed, upon the *re-*

fractive power of the plate, is easily verified by introducing a drop of water between the glasses. The rings are then observed to contract, and if we compare their diameters in air and in water, it will be found that the corresponding thicknesses of the plate are as four to three, or in the inverse ratio of the refractive indices.

We have hitherto spoken only of the reflected rings. There is another system of rings formed by *transmission*, but much fainter than the former. The transmitted rings are found to observe the same laws as the reflected rings,—with this remarkable exception, that the colour transmitted at any particular thickness of the plate is always *complementary* to that reflected at the same thickness; so that, in homogeneous light, the bright transmitted ring is always at the same distance from the centre as the corresponding dark one of the reflected system.

It is now time that we should enter on the physical account of these phenomena. For their explanation, it has been already stated, Newton invented his celebrated doctrine of the fits of easy reflexion and transmission. We have already had occasion to allude to this hypothesis in a former lecture: its application to the phenomena of thin plates is too obvious to be dwelt upon. The ray is in a fit of easy transmission in its passage through the first surface; this is succeeded by a fit of easy reflexion,—and so on alternately, the spaces traversed by the ray during the continuance of the fits, or their lengths, being all equal. On arriving at the second surface, therefore, the ray will be in a fit of easy transmission, or easy reflexion, according as the interval of the surfaces, or the thickness of the plate, is an even or an odd multiple of the length of the fit. Thus the alternate succession of bright and dark rings, and the arithmetical progression of the thicknesses at which they are exhibited, are satisfactorily explained.

To explain the second law, it is necessary to suppose that

the length of the fits varies with the colour of the light, being greatest in red light, least in violet, and of intermediate magnitude in light of intermediate refrangibility. Newton determined the absolute lengths of these fits for the rays of each simple colour, and found that they bore a remarkable numerical relation to the lengths of the chords sounding the octave. These results are even yet referred to as fundamental data in optical inquiries.

To account for the two remaining laws, Newton was constrained to make new suppositions, and to attribute properties to the fits, which are inconsistent with every physical account which has been given of them. Thus, to explain the dilatation of the rings with the obliquity, he assumed that the length of the fits augmented with the incidence, and nearly in the ratio of the square of the secant of the angle of incidence. This assumption is at entire variance with the physical theory. If the fits are produced by the vibrations of the ether which are propagated faster than the luminous molecules, and which alternately conspire with and oppose their progressive motion, their lengths should continue the same in the same medium, whatever be the incidence.

The fourth law appears to be also irreconcilable with the theory. The thicknesses of the plates of different media at which the same tint is exhibited being in the inverse ratio of the refractive indices, it was necessary to suppose that the lengths of the fits varied in the same proportion; and since, in the Newtonian theory, the refractive indices are directly as the velocities of propagation, it would follow that, as the velocities augmented, the spaces traversed by the ray in the interval of its periodical states must *diminish*, and in the same proportion.

Newton seems to have regarded this hypothesis as the mere expression of a physical fact, and in this light it was long considered. It cannot be denied that, as the thickness of the plate increases, the light appears by reflexion and

transmission *alternately*; and it is of no moment, it may be said, by what name these alternate states are called. But if we look more narrowly into the theory, we shall find that it *assumes* that the alternate appearance of the light in the reflected and transmitted pencil is the effect of an alternate reflexion and transmission at a *single* surface, that surface being the *second* surface of the plate. Now we shall find that *neither* of these suppositions is true. We shall show that there are *no alternations* of reflexion and transmission at the second surface, but that the light is *always reflected* there; and we shall further prove that the light is also reflected from the *first surface*, and that this reflexion plays an essential part in the phenomenon.

To show this, let us employ (instead of common light) light which is *polarized* in a plane perpendicular to the plane of incidence. The intensity of the light in the rings, whether of the transmitted or reflected system, varies in this case with the incidence; and the rings *vanish* altogether when the light is incident at a certain angle, which is denominated the *polarizing angle* of the glass. The same phenomenon takes place when the rings are formed by a plate of air enclosed between a lens of glass and a metallic reflector. The rings disappear, as before, at the polarizing angle of the glass; but there is this remarkable peculiarity in the phenomenon in this case:—that when the rings reappear at incidences *greater* than the polarizing angle, their intensity and colour are *complementary* to what they were before; so that, as in the one case they commence from a *black centre*, so in the other they begin from a *white* one.

To understand the conclusion to which this leads, we must assume a property of light which will be hereafter established:—namely, that when light thus polarized is incident upon glass at a certain angle, it is *wholly transmitted*, and no portion whatever reflected. This property belongs to all transparent reflecting substances, but *not* to metallic re-

flectors. In the case last mentioned, therefore, in which the rings are formed by the plate of air enclosed between a lens of glass and a metallic reflector, no light is reflected from the upper surface of the plate at the polarizing angle; but as it is abundantly reflected from the lower, or metallic surface, there seems no reason why, according to the doctrine of the fits, the rings should not still be formed. Their disappearance under these circumstances proves that the light reflected from the *upper surface* is essential to their production. There are other particulars of the experiment which show, that the light reflected from the *lower surface* also concurs in their formation; and we are thus driven to the conclusion, that the phenomena arise from the union, and mutual influence, of the pencils reflected from the two surfaces.

LECTURE VI.

COLOURS OF THIN PLATES.

GENTLEMEN,—In my last lecture I described the leading phenomena of thin plates, and stated their principal laws. I adverted also briefly to the physical theory of these phenomena advanced by Newton, and pointed out some experimental facts, observed by M. Arago, which seem to demonstrate its insufficiency. From these facts it appeared to follow that the light reflected from *both* surfaces of the plate is essential to the production of the phenomena, and we are thus led to the very threshold of the true theory.

The light incident on the first surface of the plate is in part reflected, and in part also transmitted. The transmitted portion undergoes a similar subdivision at the second surface; and part of the portion reflected at that surface will emerge through the first, and reach the eye along with that originally reflected there. Thus the reflected light consists of two portions, one reflected at the upper, and the other at the lower surface of the plate; and these two portions will *interfere*, and reinforce or weaken each other's effects, according as they reach the eye in the same, or in opposite phases.

This mode of explaining the phenomena of thin plates

was pointed out by Hooke, in a remarkable passage in his *Micrographia*, some years before the subject was taken up by Newton. In this passage he very clearly describes the manner in which the rings of successive orders depend on the interval of retardation of the second "pulse," or wave, with respect to the first; and therefore on the thickness of the plate. But he does not seem to have had any distinct idea of the principle of interference itself; and his conception of the mode in which the colours resulted from this "duplicated pulse" is entirely erroneous. Euler was the next who attempted to connect the phenomena of thin plates with the wave-theory of light; but the attempt, like all the physical speculations of this great mathematician, was signally unsuccessful, and the subject remained in this unsettled state, until the principle of interference was discovered by Young. When this principle was combined with the suggestion of Hooke, the whole mystery vanished. The application was made by Young himself, and all the principal laws of the phenomena were readily and simply explained.

Let *mon* (fig. 19) be the course of a ray reflected at the first surface of a plate; *mopo'n'* that of the ray reflected at the second surface, and twice transmitted through the first. From the point *o'* let fall the perpendicular *o'r'* upon the reflected ray *on*; it will be also perpendicular to *o'n'*, and will therefore be parallel to the front of the two reflected waves. Now let us conceive a wave reflected at the first surface, in the position *o'r'*, to meet at the same place an anterior wave reflected at the second surface, and let us calculate the original interval between them. From the time that they reached the first surface at *o*, one has travelled over the space *or'*, and the other over the space *op + po'*. But, if we let fall the perpendicular *or* upon *po'*, it is evident from the law of refraction that the spaces *or'* and *o'r* are traversed in the same time in the two media; and, consequently,

that the interval of retardation is the time of describing $op + pr$. Now $pr = op \cos 2opq$, and therefore $op + pr = op(1 + \cos 2opq) = 2op \cos^2 opq$. But $op \cos opq = pq$; and, consequently, the interval is $2pq \cos opq$. Or, if we denote that interval by δ , the thickness of the plate (pq) by t , and the angle opq by θ ,

$$\delta = 2t \cos \theta.$$

Now the two waves are in complete accordance or discordance, when the interval of retardation is an exact multiple of the length of half a wave: i. e. when

$$\delta = n \frac{\lambda}{2},$$

n being any number of the natural series. Equating these values of δ , therefore, we have, for the values of the thickness of the plate which will produce a complete accordance or discordance of the two waves,

$$t = \frac{1}{4} n \lambda \sec \theta.$$

We learn then, 1st, that the successive thicknesses of the plate, for which the intensity of the reflected light is greatest or least, are as the numbers of the natural series: 2ndly, that, for different species of simple light, these thicknesses are proportional to the length of the wave: 3rdly, that, for different obliquities, they vary as the secant of the angle of incidence on the exterior medium: and, 4thly, that, for plates of different substances, they are proportional to λ , and therefore in the direct ratio of the velocity of propagation, or in the inverse ratio of the refractive index of the substance of which the plate is composed.

There is one part of the preceding explanation which will require a little consideration. The two waves being in complete *accordance* when the interval of retardation is an *even* multiple of the length of half a wave, and in complete *discordance* when that interval is an *odd* multiple of the same

quantity, it would seem, from the foregoing account, that the bright rings should be formed at all those points for which n is an even number in the formula above given, (or the thickness an even multiple of $\frac{1}{2} \lambda \sec \theta$), and the dark rings at those points for which it is odd. If this were true, the point of contact should be a point of accordance, and the rings should commence from a bright centre, instead of a dark one.

This apparent discrepancy is explained by the fact, that the two reflexions take place under opposite circumstances; one of the rays being reflected at the surface of a *denser*, and the other at that of a *rarer* medium.

The effect of this difference will be best understood by a simple illustration. When one elastic ball strikes another at rest, it communicates motion to it in all cases; but its own condition after the shock will depend on the relative masses of the two balls. If the balls be equal, the first will remain at rest after the shock. If they be unequal, it will move; and its motion will be *in the direction* of its former motion, when its mass *exceeds* that of the second ball,—it will be in the *opposite direction* when it is *less*. This will help us to understand what passes when a wave reaches the surface separating two media. The particles of ether next the bounding surface communicate motion to the adjacent particles of the second medium, and thus give rise to the *refracted wave*. But the former particles will not remain at rest afterwards, unless the density and elasticity of the ether be the same in the two media. When this is not the case, the particles of the first medium will move, after communicating motion to those of the second, and in moving give rise to the *reflected wave*. Thus *refraction* is always accompanied by *reflexion*; and this reflexion is greater, the greater the difference of the densities of the ether in the two media. It appears also, from what has been said, that the *direction* of the

motions of the particles of the first medium, after they communicate motion to those of the second, will be different, according as the ether is *denser* or *rarer* in the first medium. In the former case the vibration of the particles is in the *same direction* that it was before; in the latter it is in the *opposite direction*. Thus there will be a reflected wave in both cases; but in one case this reflected wave is caused by a vibration in the *same* direction as that of the incident wave, in the other by a vibration in an *opposite* direction.

The result of this difference is obviously the same as if one of the systems of waves were to gain or lose half an undulation on the other; so that when the two waves, reflected from the two surfaces of the plate, should be in complete accordance, as far as depended on the difference of the lengths of their paths, they will actually be in complete discordance, and *vice versa*. Thus the *dark* rings will be formed where the thickness of the plate is any *even* multiple of $\frac{1}{2} \lambda \sec \theta$; and the *bright* ones where that thickness is an *odd* multiple of the same quantity; and thus the facts and the theory are reconciled.

We have spoken of another set of rings visible by transmission. These are produced by the interference of the rays *directly* transmitted through the plate with those which penetrate it after *two interior reflexions*. It follows from the preceding considerations that they must be *complementary* to those seen by reflexion; and this is observed to be the case. The extreme paleness of the transmitted rings arises from the great difference in the intensity of the interfering pencils.

The theory of thin plates, as it came from the hands of Young, laboured under an imperfection, which was, however, soon removed. It is obvious that the intensity of the two portions of light, reflected from the upper and under surfaces of the plate, can never be the same,—the light incident on the second surface being already weakened by par-

tial reflexion at the first. These two portions, therefore, could never wholly destroy one another by interference, and the intensity of the light in the dark rings could never entirely vanish, as it appears to do when homogeneous light is employed. M. Poisson was the first to point out, and to remedy, this defect in the theory. It is evident, in fact, that there must be an infinite number of partial reflexions within the plate, at each of which a portion is transmitted; and that it is the sum of all these portions, and not the two first terms of the series only, which is to be considered in the calculation of the effect. When the problem is taken up in this more general form, it is found that where the effective thickness of the plate is an exact multiple of the length of half a wave, the intensity of the reflected and transmitted lights will be the same as if it were removed altogether, and the bounding media placed in absolute contact; so that when these media are of the same refractive power, the reflected light must vanish altogether, and the transmitted light be equal to the incident.

Here then we have reached a point, with respect to which the two theories are completely opposed. According to both, a certain portion of light is reflected from the first surface of the plate. This, in the Newtonian theory, is left, in all cases, to produce its full effect,—and there should therefore be a considerable quantity of light in the dark rings;—while, in the wave-theory, it is, at certain intervals, wholly destroyed by the interference of the other portions, and the dark rings should be absolutely *black* in homogeneous light. The latter of these conclusions seems to accord with phenomena, while the former is obviously at variance with them. This is clearly shown by an experiment of Fresnel. A prism was laid upon a lens having its lower surface blackened, and a portion of the base of the prism was suffered to extend beyond the lens. The light reflected from this portion, according to the Newtonian theory, should not surpass

in intensity that of the dark rings. The roughest trial is sufficient to show that the intensity of the light in the two cases is widely different, and thus to prove that the dark rings cannot arise (as they are supposed to do in the theory of the fits) from the suppression of the second reflexion.

When a beam of light falls upon two plates in succession, some of the many portions into which it is divided by partial reflexion at the bounding surfaces, are frequently in a condition to interfere, and to give rise to the phenomena of colour.

Thus, when light is transmitted through *two parallel plates*, slightly differing in thickness, the colour is the same as that produced by transmission through a single plate, whose thickness is the *difference* of their thicknesses, and is found to be independent of the interval of the plates. This phenomenon was observed by Mr. Nicholson, and has been shown by Dr. Young to arise from the interference of two pencils, one of which is twice reflected within the first plate, and the other twice reflected in the second. It is obvious, in fact, that if t be the thickness of the first plate, and t' that of the second, the first pencil shall have traversed, in glass, the thickness $3t + t'$, and the second the thickness $3t' + t$, the spaces traversed in air being the same; so that the interval of retardation is the time of describing the space $2(t - t')$ in glass. Sir David Brewster observed a similar case of interference produced by two plates of *equal* thickness, *slightly inclined*; the thickness traversed in the two plates being altered by their inclination.

In both these cases, however, the interfering pencils are mixed up with, and overpowered by the light directly transmitted, and some contrivance is necessary to make the colours visible. The phenomena are much more obvious in the light reflected from both plates, which, on account of their inclination, is thus separated from the direct light. It is obvious, in fact, that the direct image of a luminous ob-

ject, seen through two glasses slightly inclined, will be accompanied by several lateral images, formed by 2, 4, 6, &c. reflexions. These images Sir David Brewster observed to be richly coloured. The bands of colour are parallel to the line of junction of the two glasses; and their breadth is greater, the less their inclination. The colours in the first lateral image are produced by the interference of the two pencils, *ABCDEFGH*, *ABCdefgh*, (fig. 20) into which the ray is divided at the first surface of the second plate; one of these portions being reflected externally by the second plate, and internally by the first,—while the other is reflected internally by the second, and externally by the first. The routes of these portions will obviously be equal when the plates are parallel, and differ in length only by reason of their inclination.

The two preceding cases of interference may be produced with plates of any thickness. What are termed the “colours of thick plates,” however, are phenomena of another kind, and arise in circumstances essentially different. These phenomena were first observed by Newton.

In Newton's experiment a beam of light is admitted through a small aperture, and received on a concavo-convex mirror with parallel surfaces, the hinder of which is silvered. A screen of white paper being then held at the centre of the mirror, and having a hole in the middle to let the beam pass and repass, a set of broad coloured rings will be depicted on it, similar to the transmitted rings of thin plates. The diameters of these rings vary inversely as the square roots of the thicknesses of the mirrors. When the mirror is inclined a little, so as to throw the reflected image a little to one side of the aperture, the rings are formed as before; but their centre is in the middle of the line joining the aperture and its image. About this centre is a spot which changes its appearance in a remarkable manner as the image recedes

from the aperture, being alternately dark and bright in homogeneous light; and in white light assuming every gradation of tint in rapid succession.

These phenomena have been shown to arise from the interference of the two portions of light, which are irregularly scattered in the passing and repassing of the ray through the refracting surface.

Thus, let o (fig. 21) be the aperture through which the beam is admitted, and let it fall perpendicularly on a reflecting plate at A and B . A portion of the incident light will be irregularly scattered in the passage of the ray, oAB , through the first surface of the plate; and this portion will diverge from the point A in all directions. Let AC be one of the rays which compose this scattered portion: this is reflected at the second surface of the plate in the direction CD , and emergent in DM . But the direct ray, AB , which is reflected back in BA , will again be partially scattered in repassing through the first surface. Let AM be one of the rays of this second scattered pencil, meeting the ray DM of the first at the point M ; and let us calculate their interval of retardation. The latter has traversed the space $AB + BA$ in glass, and AM in air; while the former has described the space $AC + CD$ in glass, and DM in air. The interval of retardation is therefore the time of describing $AM - DM$ in air, *plus* the time of describing $2(AB - AC)$ in glass; and it is easy to prove that the corresponding space in air is equal to

$$\frac{by^2}{a(2b + \mu a)},$$

in which a denotes the distance, oA , of the screen from the plate, b the thickness of the latter, and y the distance, oM , of any point on the screen from the aperture. Equating this to $n\frac{\lambda}{2}$, it follows that the successive bright and dark rings will be formed where

$$y^2 = n\lambda \frac{a(2b + \mu a)}{2b}.$$

When a is very great in comparison with b , as is usually the case, we have simply

$$y^2 = \frac{n\lambda\mu a^2}{2b}.$$

When the interval between two glasses is filled with different substances, such as water and air, or water and oil, in a finely subdivided state, the portions of light which have traversed them are in a condition to interfere, the interval of retardation depending on the difference of the velocities of light in the two media. Accordingly, coloured rings will be seen when a luminous object is viewed through the glasses; the rings being similar to those usually seen by transmission, but much larger. But when a dark object is behind the lenses, and the incident light somewhat oblique, the rings immediately change their character, and resemble those of the ordinary reflected system; one of the portions, in this case, being reflected, and therefore suffering a loss of half an undulation.

These phenomena were observed and explained by Young, and were denominated by him the "colours of mixed plates." Young also observed some similar phenomena of colour in an unconfined medium. Thus, when the dust of the lycoperdon is mixed with water, the mixture exhibits a green tint by direct light, and a purple tint when the light is indirect; and the colours rise in the series, when the difference of the refractive densities is lessened by adding salt to the water. The interval of retardation, in this case, depends on the magnitude of the transparent particle.

Before I conclude this review of the two theories, in their application to the laws of unpolarized light, I would observe, that *any* well-imagined theory may be accommodated to phenomena, and seem to explain them, if only we increase the number of its *postulates*, so as still to embrace each new

class of phenomena as it arises. In a *metaphysical* sense such a theory may be said to be *true*, so long as it is thus made to represent all known facts; but it is no longer a *physical* theory, whose very essence it is to ascend in simplicity, at the same time that it rises in generality;—it is “a mob” of individual laws, without connexion, order, or dependence.

These remarks seem to apply to the explanation of the phenomena of thin plates adopted in the theory of emission. These phenomena, Newton saw, could not be accounted for on the bare hypothesis of molecules shot from the luminous body, and subjected to the attractive and repulsive forces of the bodies which they approached or met in their progress. He was compelled to add a new property to light,—to endow these molecules with dispositions which seemed wholly alien to their other properties, and which could only be connected with them by assuming a material mechanism much more complicated than was at first proposed. But this was not all. *Each* of the laws of thin plates required a *new* property in the fits to which they were referred; and *none* of these properties were in any way related to the rest, or to the mechanism on which they were supposed to depend.

These imperfections of the Newtonian theory are still more glaring when we pass from one class of phenomena to another. The phenomena of *diffraction*, for example, are referred to principles altogether different from those which seemed to be required in explaining the colours of *thin plates*; and the two classes of phenomena, in this way of accounting for them, bore no relation of any kind to each other.

All this is otherwise in the wave-theory. Here all is simple—all in harmony. The principle of interference is a *necessary* consequence of the nature of a vibration; and this *one principle*, as we have seen, explains in the most complete manner the laws of the other classes of phenomena.

But it is not merely in their reference to a common origin that these phenomena are thus found to be related. They are even bound by the chain of number. The simple laws of *interference*,—the laws of *diffraction*, and those of *thin plates*, which we have seen to be reducible to them,—are all dependent upon a *single constant*,—the length of a wave of light in each medium; and this constant being inferred from any one experiment, in any one class of phenomena, we can compute *numerically* the details of all the rest, and compare them with the results of actual measurement. The agreement is found to be complete.

It is impossible to resist the conviction of the truth of a theory supported by such evidence. The explanation which it affords is not only *complete*, but it is also *simple*. Its ultimate laws, and its ultimate data, seem to be reduced to the smallest possible number; and, when we contemplate the endless variety of phenomena which are thus simply grouped together, we feel that we have indeed been permitted to advance a step in the knowledge of the works of the INFINITE ONE.

THE END.

LECTURE I.

DOUBLE REFRACTION.

GENTLEMEN,—We have hitherto assumed that, when a ray of light is incident upon the surface of a transparent medium, the intromitted portion pursues, in all cases, a single determinate direction. This is, however, very far from the fact. In many, indeed in most cases, the refracted ray is divided into *two distinct pencils*, each of which pursues a separate course, determined by a distinct law. This property is called *double refraction*. It was first discovered by Erasmus Bartholinus, in the well-known mineral called Iceland spar. After a long series of observations, he found that one of the rays within the crystal followed the known law of refraction, while the other was bent according to a new and extraordinary law not hitherto noticed by philosophers. An account of these experiments was published at Copenhagen, in the year 1669, under the title “*Experimenta Crystalli Islandici dis-diaclastici, quibus mira et insolita refractio detegitur.*”

A few years subsequent to the date of this publication, the subject was taken up by Huygens. This distinguished philosopher had already unfolded the theory which supposes light to consist in the undulations of an ethereal fluid; and from that theory had derived, in the most lucid and elegant manner, the laws of *ordinary refraction*. He was, therefore,

naturally anxious to examine, whether the new properties of light, discovered by Bartholinus, could be reconciled to the same theory; and, in his desire to assimilate the two classes of phenomena, he was happily led to assign the true law of *extraordinary refraction*. The important researches of Huygens on this subject are contained in the fifth chapter of his "*Traité de la Lumière*."

The property of double refraction is possessed by all crystallized minerals, excepting those belonging to the *tessular system*, i. e. those whose fundamental form is the cube. It belongs likewise to all animal and vegetable substances, in which there is a regular arrangement of parts; and, in fine, to all bodies whatever, whose parts are in a state of *unequal compression* or *dilatation*. The separation of the two refracted pencils is in some cases considerable, and the course of each easily ascertained by observation; but it is generally too minute to be directly observed, and its existence is only proved by the appearance of certain phenomena, which are known to arise from the mutual action of two pencils. In Iceland spar, the substance in which the property was first discovered, the separation of the pencils is very striking: and, as this mineral is found in considerable masses, and in a state of great purity and transparency, it is admirably fitted for the exhibition of the phenomena.

Carbonate of lime, of which Iceland spar is a variety, is composed of 56 parts of lime, and 44 of carbonic acid. It crystallizes in more than 300 different forms; all of which may be reduced by cleavage to the *rhombohedron*, which is accordingly the primitive form. This solid, you are aware, is bounded by six equal rhomboidal faces, or equilateral parallelograms (fig. 1). The angles of these parallelograms, CAB and ABD, in the rhombohedron of Iceland spar, are $101^{\circ} 55'$ and $78^{\circ} 5'$. Two of the solid angles, at A and o, are contained by three obtuse angles; while the remaining four are bounded by one obtuse and two acute angles. The

line AO , joining the summits of the obtuse solid angles, is called the axis of the rhombohedron, and is equally inclined to the three faces which meet there. The angles at which the faces themselves are mutually inclined are $105^{\circ} 5'$ and $74^{\circ} 55'$.

If a transparent piece of this substance be laid upon a sheet of white paper, on which a small black spot is marked with ink, we see two images of the spot instead of one, on looking through the crystal; and if the eye be held perpendicularly above the surface, and the crystal turned round in its plane, one of these images will appear to describe a circle round the other, which is immoveable, the line connecting them being in the direction of the shorter diagonal of the rhombic face. We may vary this experiment, by substituting for the dark spot on the paper a luminous point on a dark ground; as, for example, the light of the sky seen through a small aperture. But the most direct mode of performing the experiment is to transmit a ray of the sun's light through the crystal, and receive the emergent pencils on a screen.

If now we examine the course of the two rays within the crystal, we shall find that, at a perpendicular incidence, the deviation of one of them is nothing; that, at any other incidence, the ray is bent towards the perpendicular, the sines of the angles of incidence and refraction being in the constant ratio of 1.654 to 1; and that these angles are always in the same plane. This ray, therefore, is refracted according to the known law, and is called the *ordinary ray*. On examining the other ray, however, we find that, at a perpendicular incidence, the deviation, instead of vanishing, is $6^{\circ} 12'$; that, at other incidences, the refracted ray does *not* follow the law of the sines; and that, moreover, the angles of incidence and refraction are in *different planes*. This ray, therefore, is refracted according to a new and different law, and is called the *extraordinary ray*.

In proceeding to the consideration of this law, we must observe, in the first place, that there is a certain direction in

every double-refracting crystal, along which, if a ray be transmitted, it is no longer divided. This line is called the *optic axis*, and all the phenomena of double refraction are related to it. There are, properly speaking, an infinite number of such lines within the crystal, all parallel to one another ; so that the optic axis is fixed, not in position, but in *direction* only. I have already mentioned that the line connecting the obtuse solid angles of the rhombohedron of Iceland spar is the axis of the crystal. Now if we conceive a crystallized mass of this substance to be subdivided into its elementary molecules, which are of this form, the axis of each of these molecules will be an optic axis. The optic axis of the crystallized mass, therefore, is a direction in space parallel to the axes of the elementary molecules, or equally inclined to the three faces containing the obtuse solid angle.

All the phenomena of double refraction are symmetrical round this line. To see this, we have only to polish an artificial face on the crystal, perpendicular to the optic axis, and to mark the course of the refracted rays. We shall then observe, that when the ray is incident perpendicularly on this face, or in the direction of the axis, it undergoes no deviation by refraction, and the ordinary and extraordinary rays coincide ; that, for every other incidence the ray is divided, the refracted rays being both in the plane of incidence, and the deviation of the extraordinary ray being less than that of the ordinary. This deviation of the extraordinary ray (and therefore the ratio of the sines) is the same for all rays equally inclined to the axis, whatever be the azimuth of the plane of incidence. But it is found, that the ratio of the sines of incidence and refraction of the extraordinary ray is not constant, but *diminishes* as the inclination of the incident ray to the optic axis *increases* ; being least of all when the ray is perpendicular to the axis. This least value of the ratio is called the *extraordinary index* ; in Iceland spar it is 1.483.

In the preceding cases, the plane of incidence *contains*

the *optic axis*, and the extraordinary ray, we have seen, continues in that plane. This is generally true under the same circumstances, whatever be the refracting surface. To see it, we have only to look obliquely through a rhomb of Iceland spar at a point on a sheet of paper: the extraordinary image will be seen to revolve round the other, as the rhomb is turned, and will *twice* arrive in the plane of incidence,—namely, when that plane contains the optic axis. The same coincidence of the two planes occurs also when the plane of incidence is *perpendicular* to the optic axis; but in this case, the ratio of the sines of incidence and refraction of the extraordinary ray is *constant*, so that this ray then satisfies *both* the laws of ordinary refraction. This constant ratio is the extraordinary index already referred to; it is best determined by means of a prism of the crystal, having its refracting edge parallel to the optic axis.

The direction of the two refracted rays is in all cases given by the following construction. Let AC (fig. 2) be the incident ray, and CF the section of the surface of the crystal made by the plane of incidence. Let the incident ray be produced any where to B , and let BF be drawn perpendicular to it, meeting the surface in F . Let $CD : CB :: \text{sine of refraction to sine of incidence of the ordinary ray}$; and from the centre c , and with the radius CD , let the sphere DOG be described. Let the spheroid of revolution GE be described with the same centre, its axis of revolution being in the direction of the optic axis of the crystal, and equal to the diameter of the sphere, while the other axis is greater in the ratio of the ordinary to the extraordinary index. The sphere and spheroid will represent the surfaces of the *ordinary* and *extraordinary waves*. Now, if through F a line be drawn perpendicular to the plane of the diagram, and through that line there be drawn tangent planes, FO and FE , to the sphere and spheroid, the lines CO and CE drawn from the centre to the points of contact, will be in the direction of the ordinary and ex-

traordinary rays. This elegant construction was given by Huygens.

For this construction Newton substituted another, without stating the theoretical grounds on which he formed it, or even advancing a single experiment in its confirmation. In this unsatisfactory position the problem of double refraction was suffered to rest for nearly a century ; and it was not until the period of the revival of physical optics, in the hands of Young, that any new light was thrown upon the question. Young was led by the theory of waves to assume the truth of the law of Huygens ; and, at his instigation, Wollaston undertook the experimental examination, which recalled to it the attention of the scientific world, and ended in its universal admission. The French Institute soon after proposed the question of double refraction as the subject of their prize essay, and the successful memoir of Malus left no doubt remaining as to the accuracy of the Huygenian law.

We have seen that in Iceland spar the extraordinary index is less than the ordinary, and that consequently the extraordinary ray is always refracted *from the axis*. This, however, is not universally true of all double-refracting crystals. Biot discovered that there were many crystals in which the extraordinary index was *greater* than the ordinary, and in which, therefore, the extraordinary ray is refracted *towards the axis*. Crystals of this kind he called *attractive*, while those of the former were denominated *repulsive* ; the extraordinary refraction being ascribed, in the theory of emission, to an attractive or repulsive force, acting as if it emanated from the axis. Among the attractive, or (as they are sometimes called) positive crystals, are *quartz, ice, zircon* ; the repulsive or negative class is far more numerous, and includes, among others, *Iceland spar, sapphire, ruby, emerald, beryl*, and *tourmaline*. The Huygenian law applies to attractive as well as to repulsive crystals, it being observed, that in the former case the axis of revolution of the ellipsoid must be

the *greater* axis of the generating ellipse ; or, in other words, that the spheroid is *prolate* instead of *oblate*.

We have hitherto assumed that there is but one optic axis in every crystal, or one direction only along which a ray will pass without division. It was reserved for Sir David Brewster to discover that the greater number of crystals possessed *two optic axes*, and to establish a connexion between these diversities of optical character, and the varieties of crystalline form. Among the most remarkable of the crystals with two axes, may be mentioned *arragonite*, *mica*, *sulphate of barytes*, *sulphate of lime*, *topaz*, and *felspar*. The angles range in magnitude through the entire quadrant ; and they accordingly afford a new and important criterion for the distinction of mineral substances.

It will appear from the foregoing, that crystalline bodies may be divided into three grand classes, with respect to their action upon light, namely—

1. *Single-refracting crystals*.
 2. *Uniaxal crystals*, or those which have one axis of double refraction.
 3. *Biaxal crystals*, or those which have two such axes.
- Now Sir David Brewster discovered, that all the crystals of the first class, i. e. all *single-refracting* crystals, belong to the *tessular* system of Mohs ; that all *uniaxal* crystals belong either to the *rhombohedral*, or to the *pyramidal* system ; and that crystals of the third class, or *biaxal* crystals, belong to one or other of the *prismatic* systems.

These important relations bear, in a very close and definite manner, upon the proximate cause of double refraction. It has been just mentioned, that the only crystals which do not possess the property of double refraction, are those belonging to the *tessular* system, i. e. those whose fundamental form is the *cube*. Now in this, and its derived forms, we can assign three lines at right angles to one another, round which the whole figure is symmetrical ; and we may,

therefore, reasonably conclude that the density and elasticity of the crystal is the same in each of these directions, and consequently the same throughout. Again, crystals with one axis of double refraction belong either to the *rhomboidal*, or to the *pyramidal* system,—systems whose fundamental forms are the *rhombohedron* and the *straight pyramid*. In each of these forms there is *one axis of figure*, or one line round which the whole is symmetrical: and we may, therefore, reasonably assume that the elasticity of the crystal is either greater or less in this direction than in others, while it is equal in all directions at right angles to it. The axis of form is, in this case, the axis of double refraction. Finally, in the *oblique pyramid*, which is the fundamental form of the *prismatic* systems, there is no one line, or axis of figure, round which the whole is symmetrical; and it is therefore probable that the density of the crystal is unequal in all the three directions. Such crystals are found to have *two optic axes*.

We have just stated, that in uniaxal crystals the optic axis is also the *axis of form*. In biaxal crystals, it did not at first appear that the optic axes were in any manner related to the lines which bound the elementary crystal. Sir David Brewster, however, ascertained that if two lines be taken, one bisecting the acute, and the other the obtuse angle contained by the optic axes, these (together with a third line at right angles to both) are closely connected with the primitive form. We shall hereafter see that these are the fundamental lines of the physical theory, the two former being those in which the elasticity of the vibrating medium is greatest and least.

These relations between the optical properties of crystals and their external forms, are so close and intimate, that any change (however produced) in one of them, is found to be accompanied by a corresponding change in the other. We shall find, hereafter, that if the form of a crystal be altered by mechanical compression, or change of temperature, its refracting

properties undergo a corresponding change, which is always in conformity with the laws above assigned.

It was long supposed that one of the refracted rays, in every crystal, followed the ordinary law of the sines, while the other was refracted according to the Huygenian law. But Fresnel has proved, both from theory and by experiment, that this is not the case, and that in biaxal crystals, *both rays* are refracted in an extraordinary manner, and according to a *new law*. It is, in fact, a consequence of his beautiful theory, that the form of the wave, which is propagated in the interior of such a crystal, is neither a sphere nor spheroid, as in uniaxal crystals, but a curved surface of the fourth order. This surface is composed of two sheets; and if tangent planes be drawn to these, after the same manner as to the sphere and spheroid in the Huygenian law, the points of contact determine the directions of the two refracted rays.

I shall have an opportunity, in a future lecture, of returning to the theory of Fresnel; and now proceed to illustrate some of the more remarkable effects of double refraction.

If a rhomboid of Iceland spar, or any other double-refracting crystal, be placed close to a small object,—as, for example, a black spot on a sheet of paper,—it will be observed that one of the images is sensibly *nearer* than the other; and that the difference of their apparent distances changes with the thickness of the crystal, and with the obliquity of the ray.

This effect is easily accounted for. It is a well-known principle of common optics, that when an object is viewed through a denser medium bounded by parallel planes—as, for example, a cube of glass—the image is nearer to the surface than the object; the difference of their distances being to the thickness of the medium, as the difference of the sines of incidence and refraction to the sine of incidence. This interval, through which the image is made to approach, increases therefore with the refractive power of the medium; thus in water it is $\frac{1}{4}$ of the thickness, in glass $\frac{1}{3}$, and so for other

media. Now as double-refracting crystals have two refractive indices, of different magnitudes, there will accordingly be two images, at different distances from the surface. In Iceland spar, the ordinary index is greater than the extraordinary, and therefore the ordinary image is nearer than the other. The reverse is the case in positive crystals, such as quartz, in which the extraordinary refraction is the greater.

The refractions being equal at the two parallel surfaces of the rhomb, whether the refraction be ordinary or extraordinary, the two rays will emerge parallel to the incident ray, and therefore parallel to one another. The distance between them will be proportional to the thickness of the crystal, and will of course be the same at all distances from the rhomb. But if the surfaces be inclined, so as to form a prism, the deviation of the two rays will be different, and they will emerge inclined to one another; consequently the separation will increase with the distance.

Such a separation is of great use in many experiments. In order to render the divergence of the emergent pencils greatest, the prism should be cut with its edge *parallel to the optic axis*; so that the refraction may take place in a plane perpendicular to the axis. In this case the ordinary and extraordinary refractions differ by the greatest amount, and therefore the difference of the deviations of the two pencils is greatest. A double-refracting prism thus cut is usually achromatized by a prism of glass, with its refracting angle turned in the opposite way.

A much better arrangement has been suggested by Dr. Wollaston. Two prisms of the *same* substance, and of equal refracting angles, are cut in such a manner, that in one the refracting edge is *parallel* to the optic axis, and in the other *perpendicular* to it. They are then united, with their refracting angles turned in opposite directions, so as to form a parallelopiped; and the effect of this arrangement, it will presently appear, is to double the separation of the images produced by

either singly. By this duplication the weak double refraction of rock crystal is rendered very sensible.

An achromatic prism of this kind is employed in the *double image micrometer*, an ingenious and valuable instrument invented by Rochon. It consists of a telescope, in which a prism, such as we have described, is introduced between the object glass and its principal focus (fig. 3); and thus two images are formed in the principal focus, whose interval is greater or less, according to the distance of the prism from that point. When the instrument is used, the prism is moved until the two images appear in contact, and its distance from the focus is then read on a graduated scale. The two angles in this case having the same subtense, the visual angle of the object is to the deviation produced by the prism, as the distance of the prism from the focus is to the focal length. Now the divergence of the two rays is constant for a given prism, and may be determined either by calculation or experiment; consequently, the visual angle is deduced from the preceding proportion. By this instrument M. Arago has determined the apparent diameters of the planets, with a precision greater than had been before attained by any other means.

The same instrument has been also employed successfully in war, to determine the distance of an inapproachable object. Thus, if it be required to ascertain the distance of the walls of a besieged town, in order to know whether they are within the range of shot, we have only to measure by this instrument the angle subtended by a man, or any other object whose average height is known. The height of the object, divided by the tangent of the angle, is the distance required.

LECTURE II.

POLARIZATION OF LIGHT.

GENTLEMEN,—In the various phenomena which take place when a ray of light encounters the surface of a new medium, it has been supposed that the direction and intensity of the several portions into which it is subdivided, are wholly *independent of the side of the ray* ; or, that the direction and intensity will continue the same, on whatever side of the ray the surface is presented, provided that the angle and the plane of incidence continue unchanged. In other words, it was taken for granted that a ray of light had *no relation to space*, with the exception of that dependent on its *direction* ; that, around that direction, its properties were *on all sides alike* ; and that, if the ray be made to revolve round that line as an axis, the resulting phenomena would be unaltered.

Huygens was the first to observe that this was not always the case. In the course of his researches on the law of double refraction, he found that when a ray of solar light is received upon a rhomb of Iceland crystal, in any but one direction, it is *always* subdivided into *two of equal intensity*. But, on transmitting these rays through a *second* rhomb, he was surprised to observe that the two portions, into which each of them was subdivided, were *no longer equally intense* ; that their relative brightness depended on the *position* of the second

rhomb with regard to the first ; and that there were two such positions in which one of the rays *vanished altogether*.

On analyzing the phenomenon, we find that these two positions are those in which the *principal sections* of the two crystals are *parallel* or *perpendicular*. When these sections are *parallel*, the ray which has undergone ordinary refraction by the first crystal, will be also refracted ordinarily by the second ; and the ray which has been extraordinarily refracted by the first, will be also extraordinarily refracted by the second. On the contrary, when the principal sections of the two crystals are *perpendicular*, the ray which has suffered ordinary refraction by the first crystal, will undergo extraordinary refraction by the second ; and the extraordinary ray of the first, will be refracted according to the ordinary law in the second. In the intermediate positions of the two principal sections, each of the rays refracted by the first crystal will be divided into *two* by the second ; and these two pencils are in general *different in intensity*, their intensities being measured by the square of the cosine of the distance from the position of greatest intensity.

From this "wonderful phenomenon," as Huygens justly called it, it appears that each of the rays refracted by the first rhomb has acquired properties which distinguish it altogether from solar light. It has, in fact, acquired *sides* ; and it is evident that the phenomena of refraction depend, in some unknown manner, on the relation of these sides to certain planes within the crystal. Such was the conclusion of Newton. "This argues," says he, "a virtue or disposition in those sides of the rays, which answers to, and sympathizes with, that virtue or disposition of the crystal, as the poles of two magnets answer to one another."

Although the phenomenon discovered by Huygens was one of the highest interest in itself, and in its bearings of such importance in the mind of Newton, as to force him to admit the existence of properties in the rays of light which, until

then, had never been imagined, yet the result remained, for more than 100 years, a *unique fact* in science : and the kindred phenomena,—the properties which light acquires in a greater or less degree in almost every modification which it undergoes,—remained unnoticed until the beginning of the present century.

In the year 1809, while Malus was engaged in the experimental researches by which he so completely established the Huygenian law of double refraction, he happened one evening to turn a double-refracting prism towards the windows of the Luxembourg palace, which then reflected the rays of the setting sun. On turning round the prism, he was astonished to find that the ordinary image of the window nearly disappeared in two opposite positions of the prism, while in two other positions, at right angles to the former, the extraordinary image nearly vanished. Struck with the analogy of this phenomenon to that which is observed when light is transmitted through two rhomboids of Iceland spar, he was led at first to ascribe it to some property which the light had acquired in its passage through the atmosphere : but he soon abandoned this idea, and found that the property in question was impressed upon the light by reflexion at the surface of the glass.

Pursuing the subject, he was led to the important discovery, that when a ray of light is reflected from the surface of glass, or water, or any other transparent medium, at certain angles, the reflected ray acquires all the characters which had been found to belong to the light which had undergone double refraction. When received upon a rhomb of Iceland spar, or a double-refracting prism, one of the two pencils into which it is generally divided vanished in two positions of the rhomb, namely, when the principal section of the crystal is parallel or perpendicular to the plane of reflexion ; while, in intermediate positions, these pencils varied in intensity through every possible gradation.

A ray, then, may acquire *sides* or *poles*—may, in short, be *polarized*—by reflexion at the surface of a transparent medium, as well as by double refraction. The *plane of polarization* is the plane of reflexion at which the effect is produced; and it is experimentally known by its relation to the principal section of a double-refracting crystal, the ray undergoing ordinary refraction only, when the principal section is parallel to the plane of polarization.

But a polarized ray possesses many other characters. To understand these, we should observe that when a ray of light, proceeding directly from a self-luminous body, is received upon a reflecting surface at a given angle, the intensity of the reflected beam will be unaltered, whether the surface be above or below, on the right or on the left of the incident beam. The case, however, is altered, if, instead of the direct light of a self-luminous body, we submit to the same trial light which has been polarized by any of the means already alluded to. It is then no longer indifferent on what side of the ray the new surface is presented. The *inclination* of the reflected or transmitted ray will, indeed, remain unaltered, on *whatever side* the surface be presented; but its *intensity* will be very different; and a ray which is *reflected* most intensely when the new surface is presented at one side, under a certain angle, will be *wholly transmitted* when it is offered to another, all other circumstances being identical.

It is evident then, that the ray which has suffered reflexion at the first surface, in this experiment, has in consequence acquired properties wholly distinct from the original light. The latter is equally reflected in every azimuth of the plane of reflexion; while, on the other hand, the intensity of the twice-reflected ray diminishes, as the angle between the reflecting planes increases; and it vanishes altogether, and the ray is wholly transmitted, when the plane of reflexion at the second surface is perpendicular to that at the first. These sides, or poles, once acquired, are retained by the ray in all

its future course, provided it undergoes no further modification by reflexion or refraction; for, whether the plates be an inch or a mile asunder, the phenomena will be the same.

A polarized ray, then, is distinguished by the following characters:—

1. It is *not divided* into two pencils by a double-refracting crystal, in two positions of the principal section with respect to the ray; being refracted *ordinarily*, when the plane of polarization coincides with the principal section, and *extraordinarily* when it is perpendicular to it. In other cases it gives rise to two pencils, which vary in intensity according to the position of the principal section.

2. It suffers *no reflexion* at the polished surface of a transparent medium, when this surface is presented to it at a certain angle, and in a plane of incidence perpendicular to the plane of polarization; while it is *partially reflected*, when the reflecting surface is presented in other planes of incidence, or under different angles.

3. We shall find that it *cannot be transmitted* through a plate of tourmaline, whose axis is parallel to the plane of polarization; while it is transmitted with an increasing intensity, as the axis of the crystal is turned from that position.

These properties are invariably connected, so that any one of them will serve to distinguish a polarized from an unpolarized ray.

The apparatus best fitted for the exhibition of these phenomena is that devised by M. Biot. It consists of a tube, furnished at its extremities with two graduated rings, which are capable of revolving in a plane perpendicular to the axis of the tube. Each of these rings carries a plate of glass set in a frame, and held by two projecting arms. These plates are capable of revolving round a transverse axis, so that their inclination to the axis of the tube may be varied at pleasure; and a small graduated circle, attached to one of the arms, measures the inclination. The whole apparatus is connected

with a vertical pillar, by a moveable joint, so that the tube may be inclined to the horizon at any angle. In this form of the apparatus, it is arranged to exhibit the properties of polarized light dependent on reflexion; in order to show the other properties, one of the glass plates may be removed, and a double-refracting prism, or a plate of tourmaline, substituted in its place.

We have now seen that a ray of light is polarized, when it is incident upon water at an angle of 53° , or upon glass at an angle of 57° . But in pursuing his inquiries, Malus found that all other reflecting substances, the metals alone excepted, impressed upon the reflected light the same modification; and that the angle of incidence at which this effect was produced (and which is called the *angle of polarization*) was in general different for every different substance.

As yet, no connexion was perceived to exist between the polarizing angle and the other properties of the substance with regard to light. Malus, in fact, conceived that the property of polarizing light by reflexion, which different bodies possessed at different angles, was wholly independent of their other modes of action upon light. It was reserved for Sir David Brewster to show the error of this supposition, and to establish one of the most important laws in the whole range of physical optics. In the year 1811, he commenced an extensive series of experiments, with the view of determining the angles of polarization of different media, and of connecting them by a law. The result of these investigations was the simple and remarkable principle, that "*the index of refraction of the substance is the tangent of the angle of polarization.*" Hence, when the refractive index is known, the angle of polarization is inferred, and *v. v.*; and we learn from the law that this angle ranges in different substances from 45° upwards, increasing always with the refractive power.

Since the refractive index is different for the differently-coloured rays which compose solar light, it follows necessarily

that all the rays of the spectrum are not polarized at the same angle; so that if a beam of solar light be reflected successively by two glass plates, whose planes of reflexion are at right angles, the reflected beam will never be wholly extinguished, but will be coloured *red* or *blue*, according as the angle of incidence is the polarizing angle of the more, or of the less refrangible rays. This effect is, of course, more observable in the highly dispersive substances; and is particularly striking in *liver of antimony* and *specular iron*.

The law of Brewster may be presented in another form. We may say that the angle of polarization is such, that the *reflected and refracted rays form a right angle*. In fact, if this angle be denoted by π , and the corresponding angle of refraction by ρ , we have

$$\tan \pi = \mu, \text{ or } \frac{\sin \pi}{\cos \pi} = \frac{\sin \pi}{\sin \rho};$$

therefore $\cos \pi = \sin \rho$, and $\pi + \rho = 90^\circ$.

Now the angle of reflexion is equal to the angle of incidence π ; consequently the angles of reflexion and refraction are complementary, and the reflected and refracted rays are perpendicular.

The law of Brewster applies to the case of light reflected from the surface of a *rarer*, as well as that of a denser medium; and it follows from it, that the *two angles of polarization*, at the bounding surface of the same two media, are *complementary*. For the index of refraction, from the denser into the rarer medium, is the reciprocal of the index, when the light proceeds in the contrary direction; consequently the tangents of the angles of polarization in the two cases are reciprocals, and the angles themselves complementary.

It follows from this, that when a beam of light falls upon a medium bounded by parallel planes, and at the polarizing angle of the first surface, the portion which enters the me-

dium shall meet the second surface also at its polarizing angle, and be completely polarized by reflexion there. For the ray being incident upon the first surface at the polarizing angle, the angle of refraction will be the complement of the angle of incidence, and will be therefore equal to the angle of polarization at the second surface. But the surfaces being parallel, the angle of refraction at the first surface is equal to the angle of incidence at the second; the ray will therefore fall upon the second surface at its polarizing angle.

From the same principles it follows, that if several plates of glass, or of any transparent substance, be arranged parallel to one another, and a ray of light be incident upon the first surface at the polarizing angle, the several portions which reach the succeeding surfaces will meet them also at their polarizing angles, and the portions reflected at each will be completely polarized. Such a *pile of plates* is therefore highly useful as a *polarizer*; for the reflected beam is necessarily far more intense than that produced by a single surface.

It has been shown, that when a beam of light is polarized by reflexion, and is then suffered to fall upon a second reflecting surface at the polarizing angle, the intensity of the twice-reflected beam will vary with the inclination of the planes of reflexion; being *greatest* when these planes are *coincident*, and *nothing* when they are *perpendicular*. In all cases, *the intensity varies as the square of the cosine of the angle formed by the two planes of reflexion*. This law was at first conjecturally assumed by Malus; its truth has since been verified by the experiments of Arago and others.

It follows, as a consequence of this law, that a beam of common light may be considered as composed of two polarized beams of equal intensity, whose planes of polarization are perpendicular. For if I denote this common intensity,

and a the angle which the plane of reflexion makes with the plane of polarization of one of the beams, $90^\circ - a$ will be the angle which it forms with that of the other. Consequently, the intensity of the reflected light in the former of these beams will be $I \cos^2 a$, and in the latter $I \sin^2 a$; and the sum of these, or the total intensity of the reflected light, is

$$I (\cos^2 a + \sin^2 a) = I.$$

The intensity is therefore constant, and independent of the position of the plane of reflexion with respect to the ray; and this, we have seen, is the essential character of common, or unpolarized light.

We now proceed to consider the effects which take place, when the light is incident upon the reflecting surface at an angle different from the polarizing angle.

Malus observed, that when the angle of incidence was either greater or less than the polarizing angle, the properties already described were only *in part* developed in the reflected pencil; that neither of the two pencils into which it was divided by a rhomb of Iceland spar ever wholly vanished; but that they varied in intensity between certain limits, these limits being closer the more remote the incidence from the polarizing angle. From this he naturally concluded, that, in these circumstances, a *portion only* of the reflected pencil had received the modification to which he had given the name of polarization, that portion increasing as the incidence approached the polarizing angle; and that the remaining portion was unmodified, or in the state of common light. *Partially polarized* light, then, according to Malus, is composed of two portions, one of which is wholly polarized, while the other is in the state of ordinary, or unpolarized light. In this supposition Malus has been followed by most subsequent philosophers. A different view of the phenomenon of partial polarization has been taken by Sir David Brewster, to which I shall have occasion hereafter to allude.

If this partially-polarized light be reflected at a second surface in the same plane, and at the same angle, the reflected pencil is found to contain a greater proportion of polarized light; and by increasing the number of successive reflexions, the light may become, as to sense at least, wholly polarized. This fact was discovered by Sir David Brewster; and it was found that light may be polarized *at any incidence*, by a sufficient number of reflexions,—the number of reflexions necessary to produce this result increasing, as the incidence is more removed from the polarizing angle.

It remains to say a few words of the modification which light undergoes in *refraction*.

When a beam of common light is suffered to fall upon a plate of glass, a portion of it in all cases enters the plate, and is refracted; and this refracted portion is found to be *partially polarized*. The quantity of polarized light in the refracted beam increases with the incidence, being nothing at a perpendicular incidence, and greatest when the incidence is most oblique. The plane of polarization is not (as in the case of reflected light) coincident with the plane of incidence, but perpendicular to it.

The connexion between the light thus polarized, and that polarized by reflexion, is very intimate, the two portions being always of equal intensity. This remarkable law was discovered by Arago, and may be thus enunciated—“*When an unpolarized ray is partly reflected at, and partly transmitted through, a transparent surface, the reflected and transmitted portions contain equal quantities of polarized light; and their planes of polarization are at right angles to each other.*” Accordingly, when a beam of light falls on the surface of a transparent medium, a portion of it is always subdivided into two of equal intensity, one polarized in the plane of incidence, and the other in the perpendicular plane; and the former of these portions is reflected, while the latter is refracted.

If the transmitted beam be received upon a second plate parallel to the first, the portion of common light which it contains undergoes a new subdivision; and so continually, whatever be the number of plates. Hence, when that number is sufficiently great, the transmitted light will be, as to sense, *completely polarized*. No one can fail to observe the analogy between this phenomenon and the polarization of a ray by successive reflexions; there is, however, this remarkable difference between the two cases, namely, that the plane of polarization of the refracted ray does not (like that of the reflected ray) coincide with the plane of incidence, but is perpendicular to it. These facts were discovered by Malus. The laws of the phenomena have been since investigated, in much detail, by Sir David Brewster; and he has drawn the conclusion, that when a ray of light is transmitted through any number of plates, in the same plane of incidence, the polarization will be (as to sense at least) complete, when the sum of the tangents of the angles of incidence is equal to a certain constant. Hence, when the plates are parallel, and the incidence therefore the same on all, the tangent of the angle of complete polarization is inversely as their number.

It is a remarkable consequence of these principles, that when a ray is incident upon a pile of parallel plates at the polarizing angle, after passing a certain number, the *intensity* of the transmitted beam will undergo *no further diminution*. For, as soon as the transmitted beam is wholly polarized, since its plane of polarization is perpendicular to the plane of incidence, no portion of it whatever will be reflected by any of the succeeding plates; and it is therefore transmitted without diminution through them, whatever be their number. The case is different, however, when the light is incident on the pile at any other than the polarizing angle; and it is therefore manifest that the intensity of the light transmitted through a thick pile is greatest, when it is incident at the polarizing angle.

There are certain crystals which, like the pile of transparent plates, possess the property of polarizing the transmitted light—a property which has been by some thought to depend upon the same principles. Thus, when a ray of common light is suffered to pass through a plate of *tourmaline*, it emerges *polarized in a plane perpendicular to the axis of the crystal*. This fact may be proved by submitting the ray to any, or all, of the three tests of polarization already mentioned.

This property depends upon a peculiarity in the absorbing powers of double-refracting crystals,—namely, that the *absorption of a polarized ray varies with the position of its plane of polarization* with respect to the crystal. Tourmaline, for example, absorbs a polarized ray more rapidly when the plane of polarization is parallel to the axis, than when it is perpendicular to it. But when a ray of *common light* falls upon this crystal, it is divided into two, one of which is polarized in a plane passing through the axis, and the other in a plane perpendicular to it. Accordingly, the former of these is absorbed more rapidly than the latter; and, when the plate is sufficiently thick, the latter portion alone will be sensible in the transmitted light.

The tourmaline, accordingly, is of the greatest service in experiments on polarized light; not only as affording a ready test of polarization, but also in producing a polarized beam. It has the disadvantages, however, that the polarization of the emergent beam is never perfect, and that the intensity of that beam is much weakened by absorption—both the rays being absorbed in their passage through the crystal, though with unequal energies. The polarization produced by double refraction is the most complete of any; while the intensity of the polarized beam is greater than that produced by any other means, being very nearly *one-half* of the intensity of the original beam. The intensity of the light reflected from a plate of glass, at the polarizing angle, is not more than the $\frac{1}{12}$ th part of that of the incident light.

LECTURE III.

POLARIZATION OF LIGHT.

GENTLEMEN,—Having examined the principal phenomena of polarized light, and their laws—so far as these laws can be detected by experiment—we may now proceed to consider their connexion with the physical theory.

On a former occasion, when considering the phenomena of unpolarized light, I discussed their bearing upon the *two* theories—the theory of Newton and that of Huygens; and it appeared, on an examination of the facts then considered, that the weight of probability was altogether in favour of the latter. Of the phenomena which we have been lately considering, and of those which still remain to be unfolded, the explanations afforded by the Newtonian theory are, comparatively speaking, so scanty and incomplete, that we shall best consult our own progress in this interesting branch of science, by confining our attention in what follows to the wave-theory.

It is strange that the department of optics in which the wave-theory now stands unrivalled, should be the very same which Newton selected as affording the most decisive evidence against it. “Are not,” says he, “all hypotheses erroneous, in which light is supposed to consist in pressure, or motion, propagated through a fluid medium?” . . . “for pressures, or

motions, propagated from a shining body through an uniform medium, must be on all sides alike; whereas it appears that the rays of light have different properties in their different sides." In this objection Newton seems to have had his thoughts fixed upon that species of undulatory propagation, whose laws he himself had so sagaciously unfolded. When *sound* is propagated through *air*, or *water*, or any other uniform medium, the vibrations of the particles of the air, or water, are performed in the same direction in which the wave advances; and if the vibrations of the ether which constitute light were of the same kind, the objection would seem to be insuperable. For, if the particles of the ether be supposed to vibrate in the direction of the ray itself, it seems inconceivable that such a ray could bear a different relation to the different parts of the surrounding space; every thing, in fact, would be similar on all sides of the ray—above and below, on the right hand and on the left.

But the case is altered, if the vibrations of the ethereal particles be performed in a transverse direction. Let us suppose that the direction of these vibrations is *perpendicular* to that of the ray: then it is obvious that if these directions be *up* and *down* (for example) while the ray advances *horizontally*, such a ray will bear a relation to the parts of space *above* and *below* different from that which it bears to those parts which are on the *right hand* and on the *left*. Such is, in fact, the mode of vibration which is now assumed to belong to the ether, in the wave-theory,—the vibrations of the ethereal molecules being supposed to be performed *in the plane of the wave*; and we shall find that, with the help of this assumption, all the complicated phenomena of polarization and double refraction are explained in the fullest and most complete manner.

The principle of *transversal vibrations*, as it is called, first suggested itself to the sagacious mind of Young. Fresnel arrived at the same principle independently, and has

reared upon its basis the noblest fabric which has ever adorned the domain of physical science, Newton's system of the universe alone excepted.

When an ethereal molecule is displaced from its position of equilibrium, the forces of the neighbouring molecules are no longer balanced, and their resultant tends to drive the particle back to its position of rest(*a*). The displacement being supposed to be very small, in comparison with the intervals between the molecules, the force thus excited will be proportional to the displacement; and from this it follows, according to known mechanical principles, that the trajectory described by the molecule will be an ellipse, whose centre coincides with the position of equilibrium. Hence the vibration of the ethereal molecules is, in general, *elliptic*, and the nature of the light depends on the *direction* and *relative magnitude of the axes*. By the principle above referred to, these elliptic vibrations are all *in the plane of the wave*; their axes, however, may either preserve constantly the *same direction* in that plane, or they may be continually shifting. In the former case, the light is said to be *polarized*; in the latter, it is unpolarized, or common light.

The relative magnitude of the axes of the ellipse determines the nature of the polarization. When the axes are *equal*, the ellipse becomes a *circle*, and the light is said to be *circularly polarized*. On the other hand, when the lesser axis vanishes, the ellipse becomes a *right line*, and the light is said to be *plane-polarized*—the vibrations being in this case confined to a single plane passing through the front of the wave. In intermediate cases, the polarization is called *elliptical*, and its character may vary indefinitely between the two extremes of plane polarization and circular polarization. For the present we shall confine our attention to the case of right-lined vibrations.

(a) This is not strictly true, except in homogeneous or uncrystallized media.

In order to conceive the manner in which an undulation may be propagated by transversal vibrations, let us imagine a cord stretched in a horizontal position, one end being attached to a fixed point, and the other held in the hand. If the latter extremity be made to vibrate, by moving the hand up and down, each particle of the cord will, in succession, be thrown into a similar state of vibration; and a series of waves will be propagated along the cord with a uniform velocity. The vibrations of each succeeding particle of the cord, being similar to that of the first, will all be performed in the same plane, and the whole will represent the state of the ethereal particles in a *polarized* ray. The plane of vibration is the *plane of polarization*.

Now if, after a certain number of vibrations in the vertical plane, the extremity of the cord be made to vibrate in some other plane, and then in another,—and so on, in rapid succession,—each particle of the cord will, after a certain time proportional to its distance from the extremity, assume in succession all these varied vibrations; and the whole cord, instead of taking the form of a curve lying all in *one plane* (as in the last case) will be thrown into a species of *helical curve*, depending on the nature of the original disturbance. Such is the condition of the ethereal particles in a ray of common, or *unpolarized light*.

When, therefore, we admit a connexion to subsist among the particles of the ether, such as that which holds among the particles of the cord, we see that there is no difficulty in conceiving how a vibration may be propagated in a direction perpendicular to that in which it is executed. It is true, the particles of the ether are not chained together by cohesive forces, like those of the cord we have been considering; but the attractive forces which subsist among them are of the same kind, and may be shown to produce a similar effect. In fact, let us consider the ethereal fluid in the point of view in which it is regarded in the sounder physics of the present day,—let

us conceive it to be composed of molecules, separated by intervals which are probably considerable as compared with their magnitude, and acting on one another according to some law varying with the distance. When any row or line of such molecules is similarly displaced, and through a space which is small compared with the separating intervals, the molecules of the succeeding row will be moved in the same direction by the forces which are thus developed with the change of distance; so that the vibrations of the particles composing the first row will be communicated to those of the second, and thus the vibratory motion will be propagated in a direction perpendicular to that in which it takes place. The rapidity of the propagation will depend on the magnitude of the force developed by the displacement.

To account for the fact, that there are no sensible vibrations in a direction normal to the wave, we have only to suppose the repulsive force between the molecules to be very great, or the resistance to compression very considerable. For, in this case, it will be seen, the force which resists the *approach of two strata* of the fluid is much greater than that which opposes their *sliding* on one another.

But the existence of transversal vibrations, and of transversal vibrations only, is a necessary consequence of the laws of interference of polarized light, if the theory of waves be admitted at all. It has been *experimentally* proved, by Fresnel and Arago, that two rays *oppositely polarized* compound a single ray whose intensity is *constant*, whatever be the phases of vibration in which these rays meet. But *theory* shows, that the intensity of the light resulting from the union of two rays oppositely polarized will be constant, and independent of the phase, *only when the vibrations normal to the wave are evanescent*.

This conclusion is easily extended to the case of common, or unpolarized light. In unpolarized light, therefore, as in polarized, the vibrations are only on the surface of the wave;

and we must conceive such light to consist of a rapid succession of systems of waves, in which the vibrations take place in every possible direction tangent to the surface of the wave. The phenomenon of polarization consists simply in the resolution of these vibrations into two sets, in two rectangular directions, and the subsequent separation of the two systems of waves thus produced. When the resolved vibrations are not separated, but one of them is diminished in any ratio, the light is said to be partially polarized.

We may now proceed to consider the application of the principle of transversal vibrations to the problem of reflexion and refraction.

The *direction* of the light reflected and refracted at the surface of a uniform medium, is a simple consequence of the theory of waves; and, on a former occasion, I drew your attention to Huygens's demonstration of the laws of ordinary reflexion and refraction,—a demonstration which holds good whatever be the magnitude and direction of the propagated vibration, or, in other words, whatever be the *intensity* and *plane of polarization* of the light. The problem which we have now to consider is that which proposes to determine the latter quantities, or to deduce the intensities and planes of polarization of the reflected and refracted pencils, those of the incident pencil being given.

The solution of this important problem was obtained by Fresnel, although (we have now reason to think) from incomplete and even erroneous principles. In the attempt to generalize the problem, and to apply it to reflexion and refraction at the surfaces of crystallized as well as ordinary media, Professor Mac Cullagh, and Mr. Neumann of Königsberg, were led to modify and correct the principles of Fresnel. The first of these corrected principles is that denominated by Professor Mac Cullagh the *equivalence of vibrations*. It is simply this—the *resultant of the vibrations is the same in the two*

media ; and in the case we are at present considering, where there is but *one* refracted pencil, *the refracted vibration is the resultant of the incident and reflected vibrations*. The second principle required in the solution of the problem, is the well-known law of the *vis viva*, from which it is inferred, in the present instance, that the masses of ether put in motion, multiplied by the squares of the amplitudes of vibration, are the same before and after reflexion. Before we can apply these principles, however, we must know the relative densities of the ether in the two media, and the direction of the vibrations in a polarized ray, relatively to the plane of polarization. Here it is assumed, by the authors above mentioned, that *the density of the ether is the same in all bodies as in vacuo* ; and that, in polarized light, *the vibrations are parallel to the plane of polarization*. From these hypotheses, combined with the principles already mentioned, the solution is derived with great facility.

When the light is polarized *in the plane of incidence*, the principle of equivalent vibrations is, alone, sufficient to determine the magnitudes of the reflected and refracted vibrations. For, the directions of the vibrations being here in the plane of incidence, and perpendicular to the rays, these directions are necessarily inclined to one another at the same angles as the rays themselves. These angles therefore are

$$2\theta, \theta - \theta', \theta + \theta',$$

θ and θ' denoting the angles of incidence and refraction. Hence, if v and v' denote the amplitudes of the reflected and refracted vibrations, that of the incident vibration being taken as unity, we have

$$v = \frac{\sin(\theta - \theta')}{\sin(\theta + \theta')}, \quad v' = \frac{\sin 2\theta}{\sin(\theta + \theta')}.$$

When the light is polarized *perpendicularly to the plane of incidence*, the vibrations in the incident, reflected, and refracted pencils, are all parallel. The law of equivalent vi-

brations therefore gives, in this case, the following simple relation among them,

$$1 + v = v';$$

and another relation is required, in order to deduce the values of v and v' . This second relation is furnished by the principle of the *vis viva*, and is

$$m(1 - v^2) = m'v'^2,$$

m and m' denoting the masses of the ether in motion in the two media. Eliminating between these equations, we find

$$v = \frac{m - m'}{m + m'}, \quad v' = \frac{2m}{m + m'};$$

expressions which are remarkable as being identical with those for the velocities of two elastic balls after impact.

Let BA, AC , (fig. 4), represent the velocities and directions of the incident and refracted rays; AA' the separating surface of the two media; and BB', CC' , lines parallel to that surface. Then the masses of ether in motion in the two media are to one another as the parallelograms $A'B, A'C$; that is,

$$m : m' :: AB \sin A'AB : AC \sin A'AC \\ :: \sin \theta \cos \theta : \sin \theta' \cos \theta' :: \sin 2\theta : \sin 2\theta'.$$

Substituting this ratio, therefore, in the expressions for v and v' ,

$$v = \frac{\tan(\theta - \theta')}{\tan(\theta + \theta')} \quad v' = \frac{\sin 2\theta}{\sin(\theta + \theta') \cos(\theta - \theta')}.$$

The *intensity* of the light is measured by the *vis viva*, or by the mass of the ether put in motion, multiplied by the square of the amplitude of the vibration. Hence the intensities of the incident, reflected, and refracted rays, will be m , mv^2 , and $m'v'^2$, respectively;—or, if we take the intensity of the incident light as unity,

$$1, v^2, \text{ and } 1 - v^2;$$

since, by the law of the *vis viva*, $m(1 - v^2) = m'v'^2$.

Confining our attention for the present to the reflected vi-

bration, it will be seen that its amplitude, and consequently the intensity of the light, increases with the incidence, whether the light be polarized in the plane of incidence, or in the perpendicular plane,—being least when the light is incident perpendicularly, and greatest when it is most oblique. In the former case, i. e. when $\theta = 0$, each of the values of v becomes $\frac{\mu - 1}{\mu + 1}$, μ being the refractive index; so that the intensity of the light reflected perpendicularly is

$$\left(\frac{\mu - 1}{\mu + 1}\right)^2.$$

This remarkable expression was first given by Young. On the other hand, when $\theta = 90^\circ$, or the ray grazes the surface, the intensity of the reflected light is equal to that of the incident; or the whole of the light is reflected, whatever be the reflecting medium.

We have seen that a ray of *common* light may be conceived to consist of two polarized rays of equal intensity, whose planes of polarization are at right angles. Now let a ray, whose intensity = 1, be incident upon the surface of a transparent medium, and let it be resolved into two, each equal to $\frac{1}{2}$, polarized respectively in the plane of incidence, and in the perpendicular plane. Each of these polarized rays will give rise to a reflected and refracted ray; so that the actual reflected and refracted rays will each consist of two portions, one polarized in the plane of incidence, and the other in the perpendicular plane. If these portions were of equal intensity, as they are in the incident light, the reflected and refracted rays would be *unpolarized*; but this, in general, is not the case.

In the case of the *reflected* beam:—the intensity of the portion polarized in the plane of incidence is

$$\frac{1}{2} \frac{\sin^2(\theta - \theta')}{\sin^2(\theta + \theta')},$$

and that of the light polarized in the perpendicular plane,

$$\frac{1}{2} \frac{\tan^2 (\theta - \theta')}{\tan^2 (\theta + \theta')} ;$$

and the whole intensity of the reflected light is their sum. Now the first thing to be observed is, that these two quantities are *unequal*; or, that the two portions of which the reflected light consists, and which are polarized in opposite planes, are *different in intensity*. Hence the reflected light will not be of the nature of common, or unpolarized light; but will always have an *excess* of light *polarized in the plane of incidence*, the former expression being always greater than the latter. This is otherwise expressed by saying, that the light is *partially polarized in the plane of incidence*. This, we know, is conformable to experience.

Again, the intensities of the two *refracted* portions are

$$\frac{1}{2} \left(1 - \frac{\sin^2 (\theta - \theta')}{\sin^2 (\theta + \theta')} \right),$$

polarized in the plane of incidence, and

$$\frac{1}{2} \left(1 - \frac{\tan^2 (\theta - \theta')}{\tan^2 (\theta + \theta')} \right),$$

polarized in the perpendicular plane. As the latter of these quantities is greater than the former, the refracted beam always contains an excess of light polarized *perpendicularly to the plane of incidence*. The quantity of polarized light is measured by the difference of the two portions,

$$\frac{1}{2} \left(\frac{\sin^2 (\theta - \theta')}{\sin^2 (\theta + \theta')} - \frac{\tan^2 (\theta - \theta')}{\tan^2 (\theta + \theta')} \right);$$

and is therefore *equal to the quantity of oppositely polarized light in the reflected beam*. Thus the experimental law of Arago is a necessary consequence of theory.

The reflected light will be *completely polarized*, when one of the portions of which it consists vanishes; for, in this case,

the whole of the reflected light will be polarized in a single plane. Now, it is easy to see that the first portion, which is polarized in the plane of incidence, can never vanish. The second part vanishes, when $\theta + \theta' = 90^\circ$; for then the denominator of the fraction becomes infinite. The reflected light then contains only the other portion, and is therefore *completely polarized in the plane of incidence*. Since, in this case, $\theta + \theta' = 90^\circ$, we have

$$\cos \theta = \sin \theta' = \frac{\sin \theta}{\mu}, \text{ and } \tan \theta = \mu;$$

i. e. *the tangent of the angle of polarization is equal to the refractive index*. Thus the beautiful law, which Brewster had inferred from observation, is deduced as an easy consequence of Fresnel's theory.

When $\theta + \theta'$ is greater than 90° ,—i. e. when the angle of incidence exceeds the polarizing angle,—the expression for the amplitude of the reflected vibration *changes sign*, the light being polarized (as before) perpendicularly to the plane of incidence. This change of sign is equivalent to an alteration of the *phase* of the reflected vibration by 180° , as the incidence passes the polarizing angle; and the circumstance explains the remarkable fact noticed by Arago,—namely, that when Newton's rings are formed between a lens of glass and a metallic reflector, the incident light being polarized perpendicularly to the plane of reflexion, the rings change their character as the incidence passes the polarizing angle of the glass, the black centre being transformed into a *white* one, and the whole system of colours becoming *complementary* to what it was before. Mr. Airy was led to anticipate this result, from a consideration of the formula; and to show that a similar change must take place in the rings formed between two transparent substances of different refractive powers, as the incidence passes the polarizing angle of either substance.

When a *polarized* beam is received on a reflecting surface at different angles, the reflected beam is still polarized, but its *plane of polarization is changed*. This movement of the plane of polarization always takes place *towards* the plane of reflexion, and its amount depends on the angle of incidence. When this angle is *nothing*, or the ray perpendicular to the reflecting surface, the plane of polarization undergoes no change whatever. As the angle of incidence increases, the movement of the plane of polarization increases; until the incidence reaches the *polarizing angle*, in which case the change produced is greatest, and the plane of polarization is brought to coincidence with the plane of reflexion. As the angle of incidence still further increases, the change of the plane of polarization diminishes; until finally it becomes nothing when the angle of incidence is 90° .

All these phenomena, and their laws, flow readily from the theory we have been considering. To see this, let the vibration of the incident ray be resolved into two, one in the plane of incidence, and the other in the perpendicular plane. If a denote the amplitude of the incident vibration, and α the angle which it makes with the plane of incidence, these resolved portions are

$$a \cos \alpha, \quad a \sin \alpha.$$

After reflexion they become, respectively,

$$a \cos \alpha \frac{\sin(\theta - \theta')}{\sin(\theta + \theta')}, \quad a \sin \alpha \frac{\tan(\theta - \theta')}{\tan(\theta + \theta')};$$

and they compound a single vibration, inclined to the plane of incidence at an angle, whose tangent is the ratio of the component vibrations. If, then, this angle be denoted by α' , we have

$$\tan \alpha' = \tan \alpha \frac{\cos(\theta + \theta')}{\cos(\theta - \theta')}.$$

The truth of this formula has been verified by the obser-

vations of Fresnel himself, and more fully since by those of Arago and Brewster.

It will be seen at once, from the formula, that $a' = a$ when $\theta = 0$, and when $\theta = 90^\circ$; and the plane of polarization is unchanged by reflexion when the incidence is perpendicular, and when it is most oblique. Again, $a' = 0$ in two cases,—namely, when $a = 0$, and when $\theta + \theta' = 90^\circ$. Accordingly, when the incident ray is polarized in the plane of incidence, the plane of polarization remains unchanged by reflexion whatever be the incidence; and again, whatever be the plane of polarization of the incident ray, it is brought to coincide with the plane of incidence by reflexion at the polarizing angle.

The change of the plane of polarization by reflexion, has led Sir David Brewster into some ingenious speculations as to the nature of partially polarized light. If common light be conceived to consist of two pencils oppositely polarized, in planes inclined 45° on either side of the plane of reflexion, the effect of reflexion, it is obvious, will be to bring each of these planes nearer to the plane of reflexion; so that the planes of polarization of the two pencils will approach each other, and form an acute angle after reflexion. *Partially polarized* light, then, according to Sir David Brewster, consists of two polarized pencils, whose planes of polarization form an *acute angle*; and no portion of it is in the condition of ordinary light.

This theory, though open to some serious objections, explains the effects of *successive reflexions* with much elegance. When light thus constituted is received upon a second reflecting surface, in the same plane of incidence, the two pencils of which it is composed will have their planes of polarization brought nearer; and so continually, until, by a sufficient number of reflexions, these planes will, as to sense, coincide with the plane of reflexion, and the resulting light will appear to be wholly polarized in that plane. When the angle of

incidence is the *same* at each reflexion,—as when a ray undergoes any number of reflexions between two parallel plates,—the angle which the planes of polarization form with the plane of reflexion, after n reflexions, is given by the formula

$$\tan a_n = \frac{\cos^n (\theta + \theta')}{\cos^n (\theta - \theta')} ;$$

from which it follows, that the angle decreases indefinitely as the number of reflexions increases, the tangents of the angles diminishing in geometric progression. The progression is more rapid, the more the angle of incidence approaches the polarizing angle, at which $\theta + \theta' = 90^\circ$.

The plane of polarization of a polarized ray is changed by *refraction*, as well as reflexion, but in an *opposite* direction, the plane of polarization being removed farther from the plane of incidence, instead of approaching it. This movement of the plane of polarization increases with the incidence; being nothing when the ray falls perpendicularly upon the refracting surface, and greatest when the incidence is most oblique. The law of the change is expressed by the following simple formula,

$$\cotan a' = \cotan a \cos (\theta - \theta') ;$$

in which a and a' denote the angles which the plane of polarization forms with the plane of incidence, before and after refraction. This law was discovered experimentally by Sir David Brewster; it is a necessary consequence of the theory already given, and is deduced by a process exactly similar to that by which we inferred the corresponding change in the plane of polarization of the reflected ray.

Now if we conceive, as before, a beam of common light as composed of two polarized beams, whose planes of polarization are perpendicular,—one being 45° to the right, and the other 45° to the left of the plane of incidence,—the

effect of refraction will be to *increase* the angles which these planes form with the plane of incidence, and therefore also the angle which they form with one another. That angle, therefore, will now be *obtuse* instead of acute, and the planes of polarization will approach to coincidence, but in an opposite direction to that in the case of reflexion. By refraction at a second, and a third surface, this angle is still further increased; until, finally, it becomes very nearly 180° . The planes then are nearly coincident, and the ray is *polarized in a plane perpendicular to the plane of incidence*. Such is the account of the effect of successive refractions, according to the views of Sir David Brewster.

When the angle of incidence exceeds the angle of *total reflexion*,—the light passing from the denser into the rarer medium,—the expressions for the intensity of the reflected light, above given, become *imaginary*. But it is obvious that, in this case, the intensity of the reflected light is simply equal to that of the incident, there being no refracted pencil. How, then, are the imaginary expressions to be interpreted? They signify, according to Fresnel, that the *periods of vibration* of the incident and reflected waves, which had been assumed to coincide at the reflecting surface, *no longer coincide* there, when the reflexion is total; or, in other words, that the ray undergoes a *change of phase* at the moment of reflexion. The amount of this change has been deduced by Fresnel, by a most ingenious train of reasoning, based upon the interpretation of imaginary formulæ. It varies with the incidence, and is different for light polarized in the plane of incidence, and in the perpendicular plane. For light polarized in any azimuth, we have only to conceive the incident vibration resolved into two, one in the plane of incidence, and the other in the perpendicular plane. The phases of these vibrations being differently altered by reflexion, the reflected vibration will be the resultant of two vibrations at right

angles to one another, and differing in phase; the amount of the difference depending upon the angle of incidence. This vibration, consequently, will be *elliptic*, and the reflected light *elliptically polarized*. When the azimuth of the plane of polarization of the incident ray is 45° , the amplitudes of the resolved vibrations will be *equal*; and if, moreover, their difference of phase is a *quarter of an undulation*, the ellipse will become a circle, and the light will be *circularly polarized*.*

Reducing his formulæ to numbers, in the case of St. Gobain's glass, Fresnel found that the difference of phase of the two portions of the reflected light amounted to *one-eighth* of an undulation, when the angle of incidence was $54^\circ 37'$. Polishing, therefore, a parallelopiped of this glass, whose faces of incidence and emergence were inclined to the other faces at these angles (fig. 5), it followed that a ray $RR'R''R'''$, incident perpendicularly on one of these sides, and

* In fact, if x and y denote the distances of the molecule of ether from its position of rest, in the two rectangular directions; a and b the amplitudes of the two vibrations; and t the time; we have

$$x = a \cos (vt - \alpha), \quad y = b \cos (vt - \beta).$$

Whence

$$a - \beta = \arccos \left(\cos = \frac{y}{b} \right) - \arccos \left(\cos = \frac{x}{a} \right);$$

and taking the cosines of both sides, and clearing the result of radicals, we find

$$\frac{y^2}{b^2} + \frac{x^2}{a^2} - 2 \cos (\alpha - \beta) \frac{xy}{ab} = \sin^2 (\alpha - \beta).$$

This is the equation of an *ellipse* referred to its centre. When the component vibrations are equal, and differ a quarter of an undulation in phase,

$$a = b, \text{ and } \alpha - \beta = 90^\circ,$$

and the equation becomes

$$y^2 + x^2 = a^2,$$

and the path described by the molecule is a *circle*.

once reflected at each of the others, at r' and r'' , would emerge perpendicularly at the remaining side, the difference of phase in the two portions of the twice-reflected ray amounting to a quarter of an undulation. If, then, the incident ray be polarized in a plane inclined at an angle of 45° to the plane of reflexion, the emergent light will be *circularly polarized*. This was found to be the case on trial, and the theory thereby verified. The parallelopiped of which we have spoken is well known under the name of *Fresnel's rhomb*; and is of essential service in all experiments relating to circular and elliptic polarization.

If the emergent ray be now made to undergo two more total reflexions, in the same plane and at the same angle, by transmitting it through a second rhomb, placed parallel to the first, it will emerge *plane-polarized*; and its plane of polarization will be inclined 45° on the other side of the plane of reflexion. In fact, the two additional reflexions increase the difference of phase of the two portions, into which the light was originally resolved, from 90° to 180° ; and we know that two equal vibrations, whose phases differ by 180° , compound a single right-lined vibration, whose direction bisects the supplement of the angle formed by their directions.

This property enables us to distinguish at once a circularly-polarized ray from a ray of common light: *it becomes plane-polarized by two total reflexions, at an angle of $54\frac{1}{2}^\circ$* . On the other hand, it is at once distinguished from plane-polarized light, by the circumstance that it is divided into two rays of equal intensity by a double-refracting crystal, whatever be the position of the plane of the principal section.

We might naturally conjecture that the effects produced by *metals* upon the reflected light would be analogous to the phenomena of total reflexion by glass and other transparent substances,—there being no refracted ray in either case. It is accordingly found that when a plane-polarized ray is incident

upon a metallic reflector, the reflected light is *elliptically polarized*; the laws of the phenomenon are, however, different from those of total reflexion. These laws have been experimentally investigated, in much detail, by Sir David Brewster: they are as yet without the pale of the physical theory.

LECTURE IV.

DOUBLE REFRACTION.

GENTLEMEN,—In our last lecture we entered on Fresnel's beautiful theory of transversal vibrations, and gave its application to some of the phenomena of *polarization*. In the present lecture, I propose to show that the same theory furnishes also a complete account of the phenomena of *double refraction*, and of the hitherto mysterious effects impressed upon light in its passage through crystals. In attempting a sketch of this part of the theory, I shall confine myself to the principles on which it is founded, and the conclusions to which it leads: the mathematical processes, which conduct us from the one to the other, are of a nature unsuited to the lecture-room; and they present difficulties which (it is to be regretted) still rear a formidable barrier before the approach to this beautiful department of physics.

I stated in a former lecture, that, shortly after the discovery of the phenomena of double refraction in Iceland crystal by Bartholinus, Huygens succeeded in embracing them in the theory of waves, by a bold and felicitous assumption. He had already shown that the form of the wave which gave rise to the ordinary refracted ray, in glass and other uncrystallized substances, was the *sphere*; or, in other words, that the velocity of undulatory propagation was the same in all direc-

tions. One of the rays in Iceland crystal, too, was found to obey the same law; and judging that the law which governed the other, though not so simple, was yet *next in simplicity*, he assumed the form of its wave to be the *spheroid*; that is, he supposed the velocity of propagation to be different in different directions, being in all cases given by the following construction: "Let an ellipsoid of revolution be described round the optic axis, having its centre at the point of incidence, and let the greater axis of the generating ellipse be to the less in the ratio of the greatest to the least index of refraction, then the velocity of any ray will be represented by the radius-vector of the ellipsoid which coincides with it in direction." We have already seen that the construction for the direction of the rays derived from this assumption was verified by experience; and we have here another interesting instance, to which the history of science affords more than one parallel, of the value of analogical principles in directing scientific research.

The law of Huygens was found to hold in many other crystals beside that to which it was originally applied; and in all of these there was found to be *one optic axis*, or one line along which a ray of light passes without division. But when the indefatigable researches of Brewster made known a class of crystalline bodies, having *two optic axes*, or two lines of no separation, Huygens's law was found not to be general; and it was ascertained that one of the rays, at least, in *biaxal* crystals, followed some new and unknown law.

In this state of the question, the problem of double-refraction was taken up by Fresnel; and by the aid of a natural and simple hypothesis, combined with the principle of transversal vibrations, he has been conducted to its complete solution:—a solution which not only embraces all the known phenomena, but has even outstripped observation, and predicted consequences which were afterwards verified by experiment.

Fresnel starts from the supposition, that the elastic force of the vibrating medium, within every crystal, is in general different in different directions. This is, in fact, the most general supposition that can be made; and whether we suppose that the vibrating medium is the ether within the crystal, or that the molecules of the body itself partake of the vibratory movement, there will be obviously such a connexion and mutual dependence of the parts of the solid and those of the medium in question, that we cannot hesitate to admit for the one, what has been already established on the clearest evidence for the other.

It is easy to see, generally, that the phenomenon of double refraction is a necessary consequence of this hypothesis, and of the principle of transversal vibrations.

Let us take, for example, the simple case of a beam of light proceeding from an infinitely distant point, and falling *perpendicularly* on the surface of a uniaxal crystal, cut *parallel to the axis*. The incident wave being plane, and parallel to the surface of the crystal, the vibrations are also parallel to the same surface; and we may conceive them to be composed of vibrations *parallel* and *perpendicular to the axis* of the crystal. Now, the elasticity brought into play by these two sets of vibrations being different, they will be propagated with different velocities; and there will be *two waves* within the crystal *oppositely polarized*. If the second face of the crystal be *parallel* to the first, the two rays will emerge perpendicularly; and the only effect produced will be, that one will be *retarded* more than the other, in its progress through the crystal. But if the second face be *oblique* to the direction of the rays, they will be both *refracted* at emergence, and *differently*; and they will therefore diverge from one another.

To return to the general theory: Let us suppose a disturbance to be produced in a medium such as we have been considering, and any particle of the medium displaced from its

position of rest. The resultant of all the elastic forces which resist the displacement, will not, in general, act in the direction of the displacement, (as would be the case in a medium *uniformly* elastic), and therefore will not drive the displaced particle directly back to its position of equilibrium. Fresnel has shown, however, that there are *three* directions, at right angles to each other, in every medium of this nature, in any of which, if the particles are displaced, the elastic forces do act *in the direction of the displacement*, whatever be the nature or laws of the molecular action; and the only supposition which he makes is, that these three directions are *parallel* throughout the crystal. In fact, the first principles of crystallization compel us to admit, that the arrangement of the molecules of the crystalline body is similar in all parallel lines throughout the crystal; and the same property must belong to the ether within it, if (as we have every reason to presume) its elasticity be dependent on that of the crystal itself. These three directions Fresnel denominates *axes of elasticity*; and he thinks that they must also be axes of symmetry, with respect to the crystalline form.

If, on each of these axes, and on every line diverging from the same origin, portions be taken, which are as the square roots of the elastic forces in their directions, the locus of the extremities of these portions will be a surface, which Fresnel denominates the *surface of elasticity*. Its equation is

$$r^2 = a^2 \cos^2 \alpha + b^2 \cos^2 \beta + c^2 \cos^2 \gamma;$$

a^2, b^2, c^2 , being the elasticities in the directions of the three axes, r the radius-vector of the surface, and α, β, γ , the angles which it makes with the axes.

This surface determines the *velocity of propagation of the wave*, when the direction of its vibrations is given. For, suppose the particle to vibrate in the direction of any radius-vector, r , of this surface:—the elastic force which governs its vibration will be proportional to r^2 ; and, since the velocity

of wave-propagation in any elastic medium is as the square-root of the elastic force, it must in this case be represented by the radius-vector of the surface of elasticity in the direction of the vibrations. Hence, if we conceive the vibration in the incident wave to be resolved into two within the crystal, performed in two determinate directions, these will be propagated with different velocities; and, as a difference of velocity gives rise to a difference of refraction, it follows that the incident ray will be divided into two within the crystal, which will in general pursue different paths. Thus, the bifurcation of a ray, on entering a crystal, presents no difficulty, provided we can explain in what manner the vibration comes to be resolved.

To see in what manner this takes place, let us conceive a plane wave advancing within the crystal. By the principle of transversal vibrations, the movements of the ethereal molecules are all parallel to the wave; but the motion of each molecule, when thus removed from its position of equilibrium, is resisted by the elastic force of the medium; and that force is, in general, oblique to the direction of the displacement. If the plane containing the direction of the force and that of the displacement, were *normal to the plane of the wave*, the force would be resolvable into two,—one perpendicular to the plane of the wave, which, by the principle of transversal vibrations, can produce no effect; and the other in the direction of the displacement itself, which will thus be communicated from particle to particle without change. But this, in general, is not the case. Fresnel has shown, however, that in all cases, the displacement may be resolved in two directions in the plane of the wave, at right angles to one another, such that the elastic force called into action by each component, will be in the plane passing through the component, and normal to the wave; and thus each component will give rise to a wave in which the direction of the vibrations is preserved, and which therefore will be propagated with a constant velocity.

The two directions above alluded to are those of the *greatest and least diameters of the section of the surface of elasticity* made by the plane of the wave ; so that if the original displacement be resolved into two, parallel to these directions, each component will give rise to a plane wave, in which the vibrations will preserve constantly the same direction. The velocity of propagation being represented by the radius-vector of the surface of elasticity in the direction of the displacement, the velocities of the two parts of the wave will be proportional to the *greatest and least diameters* of the section of the surface of elasticity, to which the vibrations are parallel. Thus it appears that an incident plane wave, in which the vibrations are in any given direction, will be resolved into *two* within the crystal ; and these will be propagated with *different velocities*, and consequently assume *different directions*.

The vibrations in these waves being *parallel to two fixed lines*,—namely the greatest and least diameters of the section of the surface of elasticity,—it follows that the two rays are *polarized*, and that their planes of polarization are at *right angles*, being the planes passing through the direction of the ray and these two lines. From this it follows, that the plane of polarization of one of the rays bisects the dihedral angle made by the two planes, which pass through the *normal to the wave*, and the *normals to the two circular sections* of the surface of elasticity ; and that the plane of polarization of the other is perpendicular. This coincides, very nearly, with the rule previously given by M. Biot, namely, that *the plane of polarization of one of the pencils bisects the dihedral angle formed by planes drawn through the ray and the two optic axes ; while that of the other is perpendicular, or bisects the supplemental dihedral angle*.

Thus we see that the two fundamental facts of crystalline refraction,—namely, the bifurcation of the ray, and the opposite polarization of the two pencils,—are completely accounted for.

Further, the amplitudes of the resolved vibrations are represented by the cosines of the angles which the direction of the original vibration contains with the two fixed rectangular directions; and, as the squares of these amplitudes measure the intensities of the two pencils, the law of Malus respecting these intensities is a necessary consequence.

The velocity of propagation of a *plane wave* in any direction being known, the *form of the wave*, diverging from any point within the crystal, may be found. For, if we conceive an indefinite number of plane waves, which, at the commencement of the time, all pass through the point which is considered as the origin of the disturbance, the *wave-surface* will be that touched by all these planes at any instant. Fresnel has given the following beautiful construction for its determination:—"Let an ellipsoid be conceived, whose semiaxes are a, b, c (the same as those of the surface of elasticity), and let it be cut by any diametral plane. At the centre of this section let a perpendicular be raised; and on this line let two portions be taken, whose lengths (measured from the centre) are equal to the greatest and least radii of the section. The extremities of these perpendiculars will be the loci of the double wave."

The form of the wave-surface being known, the directions of the two refracted rays are determined by tangent planes drawn to the two sheets of the surface, according to the construction of Huygens. Conceive three surfaces, having their common centre at the point of incidence, and representing, respectively, the simultaneous positions of three waves diverging from that point,—the first in air, which is a *sphere*; the other two within the crystal, which are the *two sheets* of the surface we have been considering. Let the incident ray be produced to meet the sphere, and at the point of intersection let a tangent plane be drawn. Through the line of intersection of this plane with the refracting surface, let two planes be drawn touching the two sheets of the refracted wave; the

lines connecting the centre with the points of contact are the directions of the two refracted rays(*a*).

It may be shown that the direction of the vibratory movement, at any point of the surface of the wave, coincides with the projection of the radius-vector upon the plane which touches the surface at that point. Hence, if perpendiculars be let fall from the centre, on the tangent planes to the two sheets of the wave-surface, the lines connecting their feet with the points of contact are the directions of the vibrations in the two rays; and therefore determine their planes of polarization. The perpendiculars themselves measure the velocities of propagation of the *waves*, while the radii-vectores represent those of the *rays*.

The equation of the wave-surface is of the fourth order; it has been thrown into the following elegant form by Sir William Hamilton,

$$\frac{a^2 x^2}{x^2 + y^2 + z^2 - a^2} + \frac{b^2 y^2}{x^2 + y^2 + z^2 - b^2} + \frac{c^2 z^2}{x^2 + y^2 + z^2 - c^2} = 0.$$

(*a*) If, in place of the ellipsoid mentioned above, we take that whose semiaxes are $\frac{1}{a}$, $\frac{1}{b}$, $\frac{1}{c}$, the three principal refractive indices of the medium, the surface derived from it by the same construction will represent the *normal slowness of the waves*, and is connected with the *wave-surface* by a remarkable relation of reciprocity. The properties of this surface lead to the following elegant construction for the directions of the refracted rays, a construction which is, in many cases, more convenient than that given above:—"With the point of incidence, as a common centre, construct the surfaces of *wave-slowness* belonging to air and to the crystal, respectively. Let the incident ray be produced to meet the sphere, which represents the normal slowness of the wave in air; and from the point of intersection let a perpendicular be drawn to the refracting surface. This will cut the surface of slowness of the refracted waves, in general, in two points. The lines connecting these points with the centre will represent the direction and normal slowness of the *waves*; while the perpendiculars from the centre on the tangent planes at the same points will represent the direction and slowness of the *rays*." This construction was given by Sir William Hamilton and Professor Mac Cullagh.

From the construction of the wave-surface, above given, it follows that there are two directions,—namely, the normals to the two circular sections of the ellipsoid,—in which the two sheets of the wave-surface have a common radius-vector, and therefore the two rays a common velocity. If ω and ω' denote the angles which the radius-vector at any point of the wave makes with these lines, v and v' the radii-vectores terminating in the two sheets of the wave-surface, the equation above given may be reduced to the following remarkable polar forms :

$$v^{-2} = a^{-2} \sin^2 \frac{1}{2} (\omega + \omega') + c^{-2} \cos^2 \frac{1}{2} (\omega + \omega'),$$

$$v'^{-2} = a^{-2} \sin^2 \frac{1}{2} (\omega - \omega') + c^{-2} \cos^2 \frac{1}{2} (\omega - \omega').$$

Since the radius-vector of the wave-surface measures the velocity of the ray in its direction, the velocities of the two rays are given by the preceding formulæ. If we subtract the latter from the former, we find (after a simple trigonometrical reduction),

$$v^{-2} - v'^{-2} = (a^{-2} - c^{-2}) \sin \omega \sin \omega'.$$

Hence the difference of the squares of the reciprocal velocities of the two rays is proportional to the product of the sines of the angles made by their common direction with the lines in which the two rays have a common velocity. In all known crystals, these lines deviate very little from the *optic axes*, or the lines in which the two parts of the *wave* have a common velocity; and thus the remarkable law, to the discovery of which M. Biot was led by analogy, and which has been also shown to flow from the constructions for the velocity given by Sir David Brewster, is a necessary consequence of Fresnel's theory.

The two sets of lines above alluded to—the lines of *single ray-velocity*, and *single wave-velocity*—are situated in the plane of the axes of greatest and least elasticity, the lines of each pair making equal angles with the axis of greatest elas-

ticity on either side. The tangents of these angles are, respectively,

$$\frac{a \sqrt{b^2 - c^2}}{c \sqrt{a^2 - b^2}}, \quad \frac{\sqrt{b^2 - c^2}}{\sqrt{a^2 - b^2}}.$$

Hence, when $b^2 = c^2$, or $b^2 = a^2$, these angles become 0, or 90° ; and the two optic axes *unite*,—coinciding in the former case with the axis of greatest elasticity, and in the latter with that of the least. In either of these cases, then, $\omega = \omega'$, and the preceding equations become

$$v^{-2} = a^{-2} \sin^2 \omega + c^{-2} \cos^2 \omega, \quad v' = c;$$

the former of which is the equation of the *ellipsoid* of revolution, and the latter that of the *sphere*. Accordingly, the wave-surface resolves itself into the sphere and spheroid of the Huygenian law; and the form of the wave in *uni-axal* crystals, which was *assumed* by Huygens, is deduced as a simple corollary from the general theory of Fresnel.

Finally, when the *three* elasticities are all equal, it will appear at once from the preceding equations that the spheroid becomes a sphere. The velocity is accordingly the same in all directions, and the law of refraction is reduced to the known law of Snellius.

I have already mentioned that, as soon as a class of double-refracting substances was discovered, possessing two optic axes, the construction of Huygens was found not to be general. It was still thought, however, that the velocity of *one* of the rays in every crystal was constant; or, in other words, that one of the rays was always refracted according to the ordinary law of the sines. According to Fresnel's theory, however, it appears that, in biaxal crystals, the velocity of neither of the rays is constant, and that the refraction of both is performed according to a new law. It was, therefore, a matter of much interest to decide this question by accurate experiment, inasmuch as the result of such decision would involve the fate of the new theory. This experimental problem

was solved by Fresnel himself, and the result, as will appear, was decisive in favour of his theory.

It was shown on a former occasion, that when light, diverging from a luminous origin, passes through two near apertures in a screen, the two beams into which it is thus divided will interfere, and produce fringes,—the central fringe being at those points where the rays have traversed equal paths. Now if two plates of glass, cut from the same plate, and of exactly the same thickness, be placed perpendicularly, one in the path of each ray, the two rays will be *equally retarded*, and the central fringe will remain undisplaced. But if, instead of glass plates, we employ plates cut in *different directions* from the same biaxal crystal,—the plates being of exactly the same thickness,—the fringes produced by the interference of the two ordinary(*a*) rays will remain still undisplaced, if the velocity of these rays is the same in the two crystals; while, on the other hand, if the velocities be different, the fringes will be shifted from their original position. On trial, the result was found to be as Fresnel had anticipated: the fringes *were displaced*; and the amount of that displacement agreed with the calculated difference of velocity, which had been previously deduced from the theory, combined with the observations of M. Biot.

A second method of trial employed by Fresnel, consisted in cutting two prisms in different directions from the same crystal of topaz, cementing them together, and grinding them thus to the same angle. The compound prism thus formed was achromatized by a prism of glass. On looking through the combination at a line of light, it was found that the ordinary image of the line was *broken* at the junction of the two prisms,—thus showing that the ray was unequally refracted in different directions.

I will now proceed to consider two remarkable cases of

(*a*) The ray whose velocity varies the least, in biaxal crystals, is sometimes, though improperly, called for distinction the *ordinary ray*.

Fresnel's theory, which have lately furnished a very striking confirmation of its truth.

If we make $y = 0$, in the equation of the wave-surface, so as to obtain its intersection with the plane of xz , the resulting equation is reducible to the form

$$(x^2 + z^2 - b^2) [a^2 x^2 + c^2 z^2 - a^2 c^2] = 0.$$

This equation is manifestly resolvable into the two following:

$$x^2 + z^2 = b^2, \quad a^2 x^2 + c^2 z^2 = a^2 c^2;$$

so that the surface intersects the plane of xz in a *circle* and *ellipse*. (Fig. 6). As these two curves have a common centre, and as the radius of the circle, b , is of intermediate magnitude to the semiaxes of the ellipse, it follows that they must intersect in four points, as is represented in the diagram.

Now, when two rays pass within the crystal in any common direction, as OAB , their velocities are represented by the radii-vectores of the two parts of the wave, OA and OB , and their directions, at emergence, are determined by the positions of the tangent planes at the points A and B . But in the case of the ray OP , whose direction is that of the line joining the centre with one of the four cusps, or intersections just mentioned, the two radii-vectores *unite*, and the two rays have the *same velocity*. There are still, however, two tangents to the plane section at the point P ; so that it might be supposed that the rays proceeding with this common velocity within the crystal, would still be divided at emergence into two, and two only, whose directions are determined by the tangent planes. This seems to have been Fresnel's view of the case. Sir William Hamilton has shown, however, that there is a *cuspidal cusp* at each of the four points just mentioned, not only in this particular section, but in every section of the wave-surface passing through the line OP ; or, more properly, that there is a *conoidal cusp* on that surface at the four points of intersection of the circle and ellipse, and consequently *an infinite number of tangent planes*,

which form a tangent cone of the second degree. Hence, a *single ray*, such as OP , proceeding within the crystal in one of these directions, should be divided into an *infinite number of rays* at emergence, whose directions and planes of polarization are determined by the tangent planes.

Again, it is evident that the circle and ellipse have *four common tangents*, such as MN ; and the planes passing through these tangents, and perpendicular to the plane of the section, are perpendicular to the optic axes of the crystal. Fresnel seems to have thought that these planes touched the wave-surface in the two points just mentioned, and in these only; and, consequently, that a single ray,—incident upon a biaxial crystal in such a manner, that one of the refracted rays should coincide with an optic axis, OM ,—will be divided into two within the crystal, OM and ON , determined by the points of contact. But Sir William Hamilton has shown that the four planes of which we have spoken touch the wave-surface, not in two points only, but in an *infinite number of points*, constituting each a small *circle of contact*; and, consequently, that a single ray of common light, incident externally in the above-mentioned direction, should be divided into an *infinite number of refracted rays* within the crystal.

Here, then, are two singular and unexpected consequences of Fresnel's theory, not only unsupported by any facts hitherto observed, but even opposed to all the analogies derived from experience. Here are two remote conclusions of that theory, deduced by the aid of the most refined analysis, and in themselves of a nature so singular, that we feel inclined at first to reject any theory of which they are the necessary consequences. They accordingly furnish a test of the truth of that theory of the most trying nature that can be well imagined; and it will be admitted that if such conclusions be verified, the theory itself may be regarded as certain.

Being naturally anxious to submit the wave-theory to this test, and to establish or disprove these, its new results, Sir

William Hamilton requested me to examine the subject experimentally. My first trials were unsuccessful, owing to the small dimensions of the crystals which I had at my disposal ; but having at length obtained from Dollond a specimen of arragonite of considerable size and purity, I was fortunate enough to prove the existence of both species of *conical refraction*.

The first case of conical refraction is that called by Sir William Hamilton *external conical refraction*, and was expected to take place, as we have seen, when a single ray passes within the crystal in the direction of either of the lines of *single ray-velocity*. These lines coincide nearly, but not exactly, with the optic axes of the crystal ; and, in the case of arragonite, contain an angle of nearly 20° . The plate of arragonite which I used, had its faces perpendicular to the line bisecting the optic axes ; consequently the lines above-mentioned were inclined to the perpendicular at an angle of about 10° on either side. Let these lines be represented by OM and ON (Fig. 7), equally inclined to the perpendicular OP. A ray of common light traversing the crystal in the direction OM or MO, should emerge in a cone of rays, as represented in the figure ; the angle of this cone depending on the relative magnitude of the three elasticities of the crystal, a^2, b^2, c^2 . In the case of arragonite this angle is considerable, amounting to 3° very nearly.

A thin metallic plate, perforated with a very minute aperture, was placed on each face of the crystal ; and these were adjusted in such a manner, that the line connecting the two apertures should coincide with the line MO, or any parallel line within the crystal. The flame of a lamp was then brought near the first surface of the crystal, and in such a position that the central part of the beam converging to the aperture should have an incidence of 15° or 16° . When the adjustment was completed, a brilliant annulus of light appeared, on looking through the aperture in the second surface (Fig. 8).

When the aperture in the second plate was ever so slightly shifted, so that the line connecting the two apertures no longer coincided with the line mo , the phenomenon rapidly changed, and the annulus resolved itself into the two separate pencils, into which a ray is usually divided.

The incident converging cone was sometimes formed by a lens of short focus, placed at the distance of its own focal length from the surface : in this case, the lamp was removed to a distance, and the plate on the first surface dispensed with. The same experiments were repeated with the sun's light ; and I even succeeded in throwing the emergent rays on a screen, so as to observe the section of the cone at various distances from its summit.

In the first experiments there was a considerable discrepancy between the results of observation and theory, both as to the magnitude of the cone, and some other circumstances of its appearance. These discrepancies were found to arise from the *sensible magnitude* of the little aperture on the second surface of the crystal, which suffered rays to pass which were inclined to the line om at small angles. Accordingly, the magnitude of the observed cone required a correction before it could be compared with the results of theory ; and measurements being taken with this view, and the results suitably corrected, the agreement of the observed and theoretical angles was found to be complete.

A remarkable variation of the phenomenon took place, on substituting a narrow linear aperture for the small circular one, in the plate next the lamp, in the first-mentioned mode of performing the experiment,—the line being so adjusted, that the plane passing through it and the aperture on the second surface should coincide with the plane of the optic axes. In this case, according to the hitherto received views, all the rays transmitted through the second aperture should be refracted doubly *in the plane of the optic axes* ; so that no part of the line should appear enlarged in breadth, on looking through

this aperture; while, according to Sir William Hamilton, the ray which proceeds in the direction om , should be refracted in *every plane*. This was accordingly found to be the case; in the neighbourhood of each of the optic axes, the luminous line swelled out, on either side of the plane of the axes, in an oval curve. This curve, it is easy to show, is the *conchoid of Nicomedes*, whose asymptot is the line on the first surface. Its variations of form, as the plane passing through the second aperture deviated from the plane of the optic axes, were highly curious and remarkable.

On examining the state of polarization of the rays which compose the emergent cone, I was surprised to observe that every ray of the cone was polarized in a different plane. On attentive examination, I discovered that these planes were connected by the following law,—namely, “*the angle between the planes of polarization of any two rays of the cone is half the angle between the planes containing the rays themselves and the axis.*” Having assured myself of the truth of this law by experiment, I was led to inquire how far it was in accordance with theory; and on examining Fresnel’s theory with this view, I was gratified to find that it led to the same result. In fact, the direction of the vibratory movement being that of the line joining the point of contact with the foot of the perpendicular, let fall from the centre upon the tangent plane, it is manifest that there must be, in this case, an infinite number of such directions, and therefore an infinite number of planes of polarization, which may be easily shown to be connected by the above law.

The other case of conical refraction, called *internal conical refraction* by Sir William Hamilton, was expected to take place when a single ray has been incident externally upon a biaxal crystal, in such a manner that one of the refracted rays may coincide with an optic axis. The incident ray in this case ought to be divided into a cone of rays within the crystal, the angle of which, in the case of arragonite, is equal to $1^{\circ} 55'$.

The rays composing this cone will be refracted at the second surface of the crystal, in directions parallel to the ray incident on the first, so as to form a small *cylinder of rays in air*, whose base is the section of the cone made by the surface of emergence (fig. 9).

The minuteness of this phenomenon, and the perfect accuracy required in the incidence, rendered it much more difficult of detection than the former. A narrow beam of light, proceeding from a distant lamp, was suffered to fall upon the crystal, and the position of the latter was altered with extreme slowness, so as to change the incidence very gradually. When the required position was attained, the two rays suddenly spread out into a continuous circle, whose diameter was apparently equal to their former interval. The same experiment was repeated with the sun's light, and the emergent cylinder received on a small screen of silver paper, at various distances from the crystal; no sensible enlargement of the section was observable on increasing the distance. The angle of this minute cone within the crystal was found to agree, within very narrow limits, with that deduced from theory; the theoretical angle being $1^{\circ} 55'$, and the observed angle $1^{\circ} 50'$.

The rays composing the cone are all polarized in different planes; and the law connecting these planes is the same as in the case of external conical refraction.

LECTURE V.

INTERFERENCE OF POLARIZED LIGHT.

GENTLEMEN,—Having, in the preceding lectures, examined the general phenomena of polarization and double refraction, and their explanation upon the principles of the wave-theory, it remains that we should consider the effects produced when polarized light is transmitted through crystalline substances. The phenomena displayed in such cases, are probably the most splendid in optics; and when it is considered, that through these phenomena an insight is afforded into the very laboratory of nature itself, and that we are thus enabled almost to view the interior structure and molecular arrangement of bodies, the subject will hardly be thought inferior in importance to any other to which your attention has been invited during the present course.

The first discoveries in this attractive region of science were made by Arago, who presented a memoir to the Institute, in the year 1811, on the colours of crystalline plates when exposed to polarized light. The subject has since been prosecuted with unremitting ardour by the first physical philosophers of Europe, and among the foremost by Biot, Brewster, and Fresnel.

We have already shown, that when a beam of light, polarized by reflexion, is received upon a second reflecting plate at

the polarizing angle, no reflexion will take place at the second surface when the plane of incidence there is perpendicular to that on the first. The first reflector, in any apparatus of this kind, is called the *polarizing plate*, and the second (for reasons which will presently appear), the *analyzing plate*. Now, if between the two reflectors we interpose a plate of any double-refracting substance, the capability of reflexion at the analyzing plate is suddenly restored, and a portion of the light is reflected, whose quantity depends on the position of the interposed crystal. The light is thus said (though improperly) to be *depolarized* by the crystal; and it was by this property that the double-refracting structure was detected by Malus in a vast variety of substances, in which the separation of the two rays was too small to be directly perceived.

In order to analyze this phenomenon, let the crystalline plate be placed so as to receive the polarized beam perpendicularly, and let it be turned round in its own plane. We shall then observe that there are two positions of the plate in which the light totally disappears, and the reflected ray vanishes, just as if no crystal had been interposed. These two positions are at right angles to one another; and they are those in which the *principal section* of the crystal *coincides with the plane of the first reflexion*, or is *perpendicular* to it. When the plate is turned round, from either of these positions, the light gradually increases; until it becomes a *maximum*, when the principal section is inclined at an angle of 45° to the plane of the first reflexion.

In these experiments the reflected light is *white*. But if the interposed crystalline plate be very thin, the most gorgeous colours appear, which vary with every change of inclination of the plate to the polarized beam. *Mica* and *sulphate of lime* are very fit for the exhibition of these beautiful phenomena, because they can be readily divided by cleavage into laminæ of almost any required thinness. If a thin plate of either of these substances be placed so as to receive the polarized beam

perpendicularly, and be then turned round in its own plane, the tint does not change, but varies only in *intensity*; the colour *vanishing* altogether when the principal section of the crystal coincides with the plane of primitive polarization, or is perpendicular to it,—and, reaching a *maximum*, when it is inclined to the plane of primitive polarization at an angle of 45° .

If, on the other hand, the crystal be fixed, and the analyzing plate turned, so as to vary the inclination of the plane of the second reflexion to that of the first, the colour will be observed to pass, through every grade of the same tint, into the complementary colour; it being always found that the light reflected in any one position of the analyzing plate is *complementary*, both in colour and intensity, to that which it reflects in a position 90° from the former. This curious relation will appear more evidently, if we substitute a double-refracting prism for the second reflector; for the two pencils refracted by the prism have their planes of polarization—one coinciding with the principal section of the prism, and the other at right angles to it, and are therefore in the same condition as the light reflected by the analyzing plate, with its plane of reflexion successively in these two positions. In this manner the complementary lights are seen together, and may be easily compared. But the accuracy of the relation stated is completely established by making these two pencils partially overlap; for, whatever be their separate tints, it will be found that the part in which they are superposed is absolutely *white*.

When laminæ of *different* thicknesses are interposed between the polarizing and analyzing plates, so as to receive the polarized beam perpendicularly, the tints are found to vary with the thickness. The colours produced by plates of the same crystal, of different thicknesses, follow, in fact, the same law as the colours reflected from *thin plates* of air; the tints *rising in the scale* as the thickness is diminished,

until finally, when this thickness is reduced below a certain limit, the colours disappear altogether, and the central space appears *black*, as when the crystal is removed. The thickness producing corresponding tints is, however, much greater in crystalline plates exposed to polarized light, than in thin plates of air, or any other uniform medium. The *black of the first order* appears in a plate of sulphate of lime, when the thickness is the $\frac{1}{3000}$ of an inch; between $\frac{1}{2000}$ and $\frac{1}{500}$ of an inch, we have the whole succession of colours of Newton's scale; and when the thickness exceeds the latter limit, the transmitted light is always *white*. The tint produced by a plate of mica, in polarized light, is the same as that reflected from a plate of air of only the $\frac{1}{400}$ th part of the thickness.

Pursuing the examination of the same subject for *oblique* incidences, M. Biot found that, in uniaxal crystals, the tint developed—or rather the number of periods and parts of a period belonging to a ray of given refrangibility—was determined by the *length of the path* traversed by the light within the crystal, and by the *square of the sine* of the angle which its direction made with the optic axis, jointly. In biaxal crystals, we must substitute for the square of the sine, the *product of the sines* of the angles which the ray makes with the two axes.

Let us now turn to the physical theory, and see in what manner it explains the appearances.

We have already seen that a ray, on entering a crystalline plate, is divided into two, differing in velocity as well as direction; or, in the language of the wave-theory, the wave incident upon the crystal is resolved into two sets of waves within it, which traverse the crystal in different directions, and with different velocities. One of these waves, therefore, will lag behind the other, and they will be in *different phases* of vibration at emergence. When the plate is thin, this *retardation* of one wave upon the other will amount only to a few undulations and parts of an undulation;

and it would therefore appear that we have here all the conditions necessary for their *interference*, and the consequent production of colour. Such was the sagacious conjecture of Young.

But here we are met by a difficulty. So far as this explanation goes, the phenomena of interference and of colour should be produced by the crystalline plate alone, and in common light, without either polarizing or analyzing plate. Such, however, is not the fact; and the real difficulty in this case is,—not so much to explain how the phenomena *are* produced, as to show why they are *not always* produced.

In seeking for a solution of this difficulty, we may observe that the two rays, whose interference is supposed to produce the observed results, are not precisely in the condition of those whose interference we have hitherto examined; they are *polarized*, and polarized in *opposite planes*. We are led then to inquire whether there is anything peculiar to the interference of polarized rays which may influence these results; and the answer to this inquiry will be found to complete the solution of the problem.

The subject of the *interference of polarized light* was examined, with reference to this question, by Fresnel and Arago, and its laws experimentally developed. It was found that two rays of light, *polarized in the same plane*, interfere and produce fringes, under the same circumstances as two rays of common light;—that when the planes of polarization of the two rays are *inclined* to each other, the interference is diminished, and the fringes decrease in intensity;—and that, finally, when the angle between these planes is a *right angle*, no fringes whatever are produced, and the rays no longer interfere at all. These facts may be established by taking a plate of tourmaline which has been carefully worked to a uniform thickness, cutting it in two, and placing one half in the path of each of the interfering rays. It will be thus found that the intensity of the fringes depends on the relative

position of the axes of the tourmalines. When these axes are *parallel*, and consequently the two rays polarized in the same plane, the fringes are best defined; they decrease in intensity, when the axes of the tourmalines are *inclined* to one another; and, finally, they vanish altogether when these axes form a *right angle*.

The non-interference of rays, polarized in planes at right angles to one another, is a necessary result of the mechanical theory of *transversal* vibrations. In fact, it is a mathematical consequence of that theory, that the intensity of the resultant light in that case is *constant*, and equal to the *sum* of the intensities of the two component lights, whatever be the phases of vibration in which they meet.

But though the intensity of the light does not vary with the phase of the component vibrations, the character of the resulting vibration will. It appears from theory, that two rectilinear and rectangular vibrations compound a single vibration, which will be also *rectilinear* when the phases of the component vibrations differ by an exact number of semi-undulations; that, in all other cases, the resulting vibration will be *elliptic*; and that the ellipse will become a *circle*, when the component vibrations have equal amplitudes, and the difference of their phases is an odd multiple of a quarter of a wave. These results of theory have been completely confirmed by experiment.

In the above-mentioned law we find the account of the fact which hitherto perplexed us,—namely, that no phenomena of interference or colour are produced, under ordinary circumstances, by the two rays which emerge from a crystalline plate,—for these rays are polarized in opposite planes; and we learn that, to produce the phenomena of colour in perfection, the planes of polarization of the two rays must be brought to coincidence by the analyzer.

Fresnel and Arago discovered, further, that two oppositely polarized rays will not interfere, even when their planes of po-

larization are made to coincide, unless they belong to a pencil, the whole of which was originally *polarized in one plane*; and that, in the interference of rays which had undergone double refraction, *half an undulation* must be supposed to be *lost or gained*, in passing from the ordinary to the extraordinary system,—just as in the transition from the reflected to the transmitted system, in the colours formed by thin plates.

The principle of the allowance of half an undulation is a beautiful and simple consequence of the theory of transversal vibrations. In fact, the vibration of the wave incident on the crystal is resolved into two within it, at right angles to one another,—one in the plane of the principal section, and the other in the perpendicular plane. Each of these must be again resolved, in two fixed directions which are also perpendicular; and it will easily appear from the process of resolution, that, of the four components into which the original vibration is thus resolved, the pair in one of the final directions must *conspire*, while in the other, at right angles to it, they are *opposed*. Accordingly, if the vibrations of the one pair be regarded as *coincident*, those of the other must *differ by half an undulation*. Hence, when the plane of reflexion of the analyzing plate coincides successively with these two positions, the colours (which result from the interference of the portions *in the plane of reflexion*, those in the perpendicular plane being not reflected) will be *complementary*.

The former of the two laws explains the office of the polarizing plate in the phenomena. To account mechanically for the non-interference of the two pencils, when the light incident upon the crystal is unpolarized, we may regard a ray of common light as composed of two rays of equal intensity, oppositely polarized(*a*), and whose vibrations are therefore per-

(a) More properly, a ray of common light must be regarded as composed of an indefinite number of rays polarized in all azimuths; so that if any two planes be assumed at right angles, there will be an equal quantity of light actually polarized in each. Ordinary light, in fact, consists of a series of systems of waves, in

pendicular. Each of these vibrations, when resolved into two within the crystal, and these two again resolved in the plane of reflexion of the analyzing plate, will exhibit the phenomena of interference. But the interval of retardation will differ by half a wave in the two cases; the tints produced will therefore be complementary, and the light resulting from their union will be of a uniform whiteness(*a*).

The preceding laws of interference being kept in mind, the reason of all the phenomena is apparent. The ray is originally polarized in a single plane, by means of the polarizing plate; it is then divided into two within the crystal, which are polarized in opposite planes; and these are finally reduced to the same plane by means of the analyzing plate. The two pencils will therefore interfere, and the resulting tint will depend on the *retardation* of one of the rays behind the other, produced by the difference of the velocities with which they traverse the crystal.

It was shown in our last lecture, that the difference between the reciprocals of the squares of the velocities, with

each of which the vibrations are different; the different systems succeeding one another so rapidly, that, in a moderate time, as many vibrations take place in any one plane, as in another at right angles to it. But the phenomena of interference, exhibited by common light, compel us also to admit (as Mr. Airy has observed) that the vibrations *do not change continually*; and that in each system of waves there are, probably, several hundred vibrations which are all similar,—although the vibrations constituting one system bear no relation to those of another, and the different systems succeed one another with such rapidity as to obliterate all trace of polarization.

(*a*) We have here supposed the resulting light to be simply the *sum* of the lights derived from each of the portions into which the original light was supposed to be resolved, without reference to their phase. The justice of this assumption depends upon the fact adverted to in the preceding note,—namely, that the two oppositely polarized portions, into which we have supposed common light to be resolved, differ in phase,—that difference continually varying. The same thing is true, therefore, of the final components; so that these must be regarded as lights proceeding from *different* sources, and compound a light equal in intensity to the sum of the components.

which the two rays traverse the crystal, is proportional to the product of the sines of the angles which their direction makes with the optic axes; so that if v and v' denote the velocities of the two rays, ω and ω' the angles which their direction makes with the two axes,

$$v^{-2} - v'^{-2} = c \sin \omega \sin \omega'.$$

But if t and t' denote the times occupied by the two rays in traversing the crystal, and θ the thickness actually traversed,—or the thickness of the plate multiplied by the secant of the angle of refraction,—

$$v^{-2} - v'^{-2} = \frac{t^2 - t'^2}{\theta^2} = \frac{t + t'}{\theta} \times \frac{t - t'}{\theta}.$$

Now the first factor of this product is very nearly constant; we have, therefore,

$$t - t' = \text{const} \times \theta \sin \omega \sin \omega';$$

or, the interval of *retardation* is proportional to the *product of the sines* of the angles which the direction of the rays makes with the two axes, and to the *thickness* of the crystal traversed, jointly. When the two axes coalesce, or the crystal becomes *uniaxal*, the retardation is proportional to the *square of the sine* of the angle which the direction makes with the axis. But the *tint* developed is measured by the interval of retardation; accordingly, the laws of the tints discovered experimentally by Biot, flow immediately from the theory.

It is plain that the light issuing from the crystal is, in general, *elliptically polarized*, inasmuch as it is the resultant of two waves, in which the vibrations are at right angles, and differ in phase. Hence, when homogeneous light is used, and the emergent beam is analyzed with a double-refracting prism, the two pencils into which it is divided vary in intensity as the prism is turned, neither, in general, ever vanishing. When, however, the thickness of the crystal is such, that the difference of phase of the two rays is an *exact number of semi-*

undulations, they will compound a *plane-polarized* ray at emergence,—the plane of polarization either coinciding with the plane of primitive polarization, or making an equal angle with the principal section of the crystal on the other side, according as the difference of phase is an even or odd multiple of half a wave. Accordingly, one of the pencils into which the light is divided by the analyzing prism, will vanish in two positions of its principal section; and it is manifest that the successive thicknesses of the crystalline plate, at which this takes place, form a series in arithmetical progression. On the other hand, when the difference of phase is a *quarter of a wave-length*, or an odd multiple of that quantity,—and when, at the same time, the principal section of the crystal is inclined at an angle of 45° to the plane of primitive polarization—the emergent light will be *circularly-polarized*. This is one of the simplest means of obtaining a circularly-polarized beam; but it has the disadvantage, that the required interval of phase can only be exact for waves of one particular length, and that, therefore, the circular polarization is perfect only for one particular colour.

We have seen that the phenomena of colour are only produced when the crystalline plate is thin. In thick plates, where the difference of phase of the two pencils contains a great many wave-lengths, the tints of different orders come to be superposed, (as in the case of Newton's rings, where the thickness of the plate of air is considerable), and the resulting light is white. The phenomena of colour may still, however, be produced in thick plates, by superposing two of them in such a manner, that the ray which has the greater velocity in the first shall have the less in the second. We have only to place the plates with their principal sections *perpendicular* or *parallel*, according as the crystals to which they belong are of the *same*, or of *opposite* denominations. Thus, if both the crystals be positive, or both negative, they are to be placed with their principal sections perpendicular; and on the other hand,

these sections should be parallel, when one of the crystals is positive and the other negative. The reason of this is evident.

Let us now consider the effects produced when a pencil of rays traverses a *uniaxal* crystal, in various directions inclined to the axis at small angles; and let us suppose, for more simplicity, that the crystalline plate is cut in a direction perpendicular to the axis.

Let ABCD (fig. 10) be the plate, and E the place of the eye. The visible portion of the emergent beam will form a cone, AEB, whose summit coincides with the place of the eye, and axis EO with the axis of the crystal. The ray which traverses the crystal in the direction of the axis, POE, will undergo no change whatever; and consequently will be reflected or not from the analyzing plate, according as the plane of reflexion there coincides with, or is perpendicular to, the plane of the first reflexion. But the other rays composing the cone will be modified in their passage through the crystal, and the changes which they undergo will depend on their inclination to the optic axis, and on the position of the principal section with respect to the plane of primitive polarization.

Let the circle (fig. 11) represent the section of the emergent cone of rays made by the second surface of the crystal; and let MM' and NN' be two lines drawn through its centre at right angles, being the intersections with the surface of the plane of primitive polarization, and of the perpendicular plane, respectively. Now the rays which emerge at any point of these lines will not be divided into two within the crystal, nor will their planes of polarization be altered; because the principal section of the crystal, for these rays, in the one case coincides with the plane of primitive polarization, and in the other is perpendicular to it. These rays therefore will be reflected, or not, from the analyzing plate, according as the plane of reflexion there coincides with, or is perpendicular to, the plane of the first reflexion. In the latter case,

therefore, a *black cross* will be displayed on the field, and in the former a *white one* (figs. 12, 13).

But the case is different with the rays which emerge at any other point, such as *L*. The principal section of the crystal for this ray, *OL*, neither coincides with, nor is perpendicular to, the plane of primitive polarization; and consequently the incident polarized ray will be divided into two within the crystal, whose planes of polarization are parallel and perpendicular to *OL*, respectively. The vibrations in these two rays are reduced to the same plane by means of the analyzing plate; they will therefore interfere, and the extent of that interference will depend on their difference of phase.

Now the difference of phase of the two rays varies with the interval of retardation. When this interval is an odd multiple of half an undulation, the two rays will be in complete discordance; and, on the other hand, they will be in complete accordance, and will unite their strength, when the retardation is an even multiple of the same quantity. We have seen that, for a given plate, the retardation is proportional to the square of the sine of the angle which the ray makes with the optic axis within the crystal. It may be easily shown that the sine of this angle is very nearly proportional to the sine of the angle *LEO* (fig. 10), which the emergent ray makes with the axis; and this latter to *LO*, the distance of the point of emergence from the centre. The retardation therefore varies as the square of the distance *LO* (fig. 11); and consequently the successive dark and bright lines will be arranged in circles, the squares of whose radii, *LO*, are in arithmetical progression (figs. 12, 13).

We have of course been speaking here of *homogeneous* light. When white or compound light is used, the rings of different colours will be partially superposed, and the result will be a series of iris-coloured rings separated by dark intervals. All the phenomena, in fact, with the exception of

the cross, are similar to those of Newton's rings; and we now see that they are both cases of the same fertile principle,—the principle of interference. These rings are exhibited even in *thick* crystals, because the difference of the velocities of the two pencils is very small for rays slightly inclined to the optic axis.

Let us now consider briefly the case of *biaxal* crystals. Let a plate of such a crystal be cut perpendicularly to the line bisecting the optic axes, and let it be interposed, as before, between the polarizing and analyzing plates. In this case, the bright and dark bands will no longer be disposed in circles, as in the former, but will form curves which are symmetric with respect to the lines drawn from the eye in the direction of the two axes. The points of the same band are those for which the *interval of retardation* of the two rays, $t - t'$, is *constant*. Now we have seen that this interval is proportional to the product of the sines of the angles which the direction of the rays makes with the optic axes; and these sines are, very nearly, as the distances of the point of emergence (measured on the face of the crystal) from the projections of the optic axes. Hence the product of these distances will be constant for all the points of the same curve. The curve formed by each band is therefore the *lemniscata* of James Bernouilli,—the fundamental property of which is, that the product of the radii-vectores, drawn from any point to two fixed poles, is a constant quantity. The exactness of this law has been verified, in the most complete manner, by the measurements of Sir John Herschel. The constant varies from one curve to another,—being proportional to the interval of retardation, and increasing therefore as the numbers of the natural series for the successive dark bands; for different plates of the same substance, the constant varies inversely as the thickness (figs. 14, 15).

The form of the *dark brushes*, which cross the entire system of rings, is determined by the law which governs the

planes of polarization of the emergent rays. There is no difficulty in showing, on the principles of Fresnel's theory, that two such dark curves, in general, pass through each pole; and that they are *rectangular hyperbolas*, whose common centre is the middle point of the line which connects the projections of the two axes.

LECTURE VI.

INTERFERENCE OF POLARIZED LIGHT.

GENTLEMEN,—In the phenomena which we have hitherto considered, the light has been polarized in a certain determinate plane,—this plane of polarization remaining unchanged during the whole future course of the ray, provided that it does not undergo any further modification by reflexion or refraction. There are certain cases, however, in which the plane of polarization of the ray is *continually changing* during its entire progress, sometimes revolving from *left to right*, (like the hands on the dial-plate of a clock), and sometimes in the *opposite* direction.

This remarkable phenomenon was first observed by Arago, in *rock-crystal*, the only known solid in which the property has been found to reside. To understand the distinction between this and other crystals,—we should observe, that when a polarized ray, of any simple colour, traverses a plate of Iceland spar, beryl, or any other uniaxal crystal, *in the direction of its axis*, it suffers no change of any kind. But when the ray traverses in the same manner a plate of *rock-crystal*, its plane of polarization is found to be altered at emergence. In some crystals of this substance, the plane of polarization is turned from *left to right*, while in others it is turned in an *opposite* direction; and the crystals themselves

are called *right-handed* or *left-handed*, according as they produce one or other of these effects.

The phenomena of *rotatory polarization* were analyzed with great diligence and success by Biot, and were reduced by him to the following general laws.

1. In different plates of the same crystal, the rotation of the plane of polarization is always proportional to the *thickness* of the plate. The same thing holds, very nearly, in plates of different crystals.

2. When two plates are superposed, the effect produced is, very nearly, the same as that which would be produced by a single plate, whose thickness is the *sum* or *difference* of the thicknesses of the two plates, according as they are of the same, or of opposite denominations.

3. The rotation of the plane of polarization is very different for the different rays of the spectrum, and increases with their refrangibility. For a given plate, the arc of rotation is *inversely as the square of the length of the wave*. Thus the arc of rotation, produced by a plate whose thickness is a *millimetre*, is $17\frac{1}{2}^{\circ}$ for the extreme red of the spectrum, 30° for the rays of mean refrangibility, and 44° for the extreme violet.

Since the rays of different refrangibilities emerge polarized in different planes, it follows that if a beam of *white* light be let fall upon the crystal, and be received after emergence upon an analyzing plate, this will reflect a portion of the light in every position of the plane of reflexion; and that portion will be beautifully coloured, the colour varying with the thickness of the crystal, and the position of the analyzing plate. For the analyzing plate will reflect the rays of different colours in different proportions, depending on the position of their planes of polarization with respect to the plane of reflexion; and the resulting colour will be a compound tint, which can be easily estimated.

The curious distinction, which was found to subsist between different specimens of rock-crystal, for a long time added to

the mystery of these singular phenomena. At length, however, Sir John Herschel discovered a connexion between the phenomena and the crystalline form. The ordinary form of the crystal of quartz is the six-sided prism terminated by the six-sided pyramid. The solid angles, formed at the junction of the pyramid and prism, are sometimes *replaced* by small secondary planes, which are oblique with reference to the original planes of the crystal; and the form of the crystal is then called *plagiedral*. In the same crystal these planes lean all in the same direction; and it is found that, when that direction is *to the right*, (the apex of the pyramid being uppermost), the crystal is *right-handed*; and that, on the contrary, it is *left-handed*, when the planes lean in the opposite way.

Sir David Brewster has since discovered that the *ame-thyst*, or violet quartz, is actually made up of *alternate layers* of right-handed and left-handed quartz. This remarkable structure may be traced in the fracture of the mineral; for the edges of the layers *crop out*, and give to the fracture the undulating appearance which is peculiar to this mineral. But the structure in question is displayed in the most beautiful manner, when we expose a plate of this substance to polarized light.

Biot and Seebeck subsequently discovered that some of the *liquids*, and even of the *vapours*, possess the same property as quartz; and impress a rotation on the plane of polarization of the intronitted ray, which is proportional to the thickness of the substance traversed. The fact is easily observed by transmitting a polarized ray through a long tube filled with the liquid, and closed at each end by parallel plates of glass; and then analyzing the emergent ray by a double-refracting prism. Among the liquids possessing this property are oil of turpentine, oil of lemon, solution of sugar in water, solution of camphor in alcohol, &c.; the first-mentioned of these liquids being right-handed, and the others left-handed. They do not lose their rotatory power by dilution with other liquids not possessing the property; and

they are found to retain it, even when raised to the state of vapour. They all possess the property in a much feebler degree than quartz; so that the ray must traverse a much greater thickness of the substance, in order to have its plane of polarization altered by the same amount. Thus a plate of rock-crystal, whose thickness is one millimetre, rotates the plane of polarization of the red ray through an arc of about 18° ; a plate of oil of turpentine, of the same thickness, turns the plane of polarization only through a quarter of a degree.

Biot found, moreover, that when two or more liquids possessing this property are mixed together, the rotation produced by the mixture is always the sum, or the difference, of the rotations produced by the ingredients, in thicknesses proportional to the volumes in which they enter the mixture, according as the liquids are of the same, or of contrary denominations; and he has even found that the same law holds good in many cases in which the liquids are chemically united. From these and other facts, he concludes that the property of rotatory polarization is inherent in the ultimate particles of bodies. A fact strongly opposed to this view is, that quartz is found to lose the property, when it loses its crystalline arrangement. Thus Sir John Herschel observed, that quartz held in solution by potash (liquor of flints) did not possess the property; and the same thing has been remarked by Sir David Brewster, with respect to fused quartz.

The phenomena of rotatory polarization, in rock-crystal, have been shown to arise from the interference of *two circularly-polarized* pencils, which are propagated along the axis with unequal velocities, one revolving from left to right, and the other in the opposite direction. A *plane-polarized* ray, in fact, is equivalent to two *circularly-polarized* rays of half the intensity, in which the vibrations are in opposite directions. When a plane-polarized ray, therefore, is incident perpendicularly upon a plate of rock-crystal, cut perpendicularly to the axis, it may be resolved into two such circularly-polarized rays. These are supposed to

be transmitted with different velocities; so that, when they assume a common velocity at emergence, one of them will be in advance of the other. They then compound a single ray, polarized in a single plane; and this plane, it can be shown, is removed from the plane of primitive polarization, by an angle proportional to the interval of retardation, and therefore proportional to the thickness of the crystal.

Thus the laws of rotatory polarization are completely explained; and it only remained to prove the truth of the assumption, that two circularly-polarized pencils, whose vibrations are in opposite directions, are actually transmitted along the axis of quartz with different velocities. This supposition is easily put to the test of experiment; since such a difference of velocity must produce a difference of refraction, when the surface of emergence is oblique to the direction of the ray. According to this hypothesis, therefore, a polarized ray transmitted through a prism of rock crystal, in the direction of the optic axis, should undergo *double refraction* at emergence; the two pencils into which it is divided being *circularly-polarized*. This has been completely verified by Fresnel, by means of an achromatic combination of right-handed and left-handed prisms, arranged so as to double the separation; and he has shown that the two pencils are neither *common* nor *plane-polarized* light, but possess all the physical characters of light *circularly-polarized*.

The subject of the peculiar refraction of rock-crystal was next taken up by Mr. Airy; and he succeeded in showing that when a plane-polarized ray is transmitted through this substance, in any direction *inclined to the axis*, it is divided into two pencils which are *elliptically-polarized*; the elliptical vibrations in the two rays being in opposite directions, and the greater axes of the ellipses coinciding respectively with the principal plane, and with the perpendicular plane. The ratio of the axes, in these ellipses, varies with the inclination of the ray to the optic axis,—being a ratio of equality when

the direction of the ray coincides with the axis, and increasing indefinitely with its inclination to that line. With respect to the *course* of the refracted rays, Mr. Airy found that it was still determined by the Huygenian law; but that the sphere and spheroid, which determine the velocity and direction of the two rays, *do not touch*, as in all other known uniaxal crystals,—the latter surface being contained *entirely within* the former. This is a necessary consequence of the fact, that the interval of retardation of the two pencils does not vanish, when the inclination of the ray to the optic axis becomes nothing.

Mr. Airy has given an elaborate calculation, founded on these hypotheses, of the forms of the rings, &c. displayed by quartz in plane-polarized, and circularly-polarized light; and he has found the most striking agreement between the results of calculation and experiment. Among the most remarkable of the phenomena whose laws are thus developed, is that produced by the superposition of two plates of quartz, of the same thickness, one of them being right-handed and the other left-handed.

All these complicated facts have been linked together, and their laws deduced by Professor MacCullagh. In this investigation, which is contained in the seventeenth volume of the Transactions of the Royal Irish Academy, the author sets out by assuming the form of the differential equations of vibratory motion in rock-crystal; and from this form he has deduced the elliptical polarization of the two pencils,—the law of the ellipticity as depending on the inclination of the ray to the axis,—the interval of retardation in the direction of the axis,—and the peculiar form of the wave-surface. It is a remarkable confirmation of this theory, that it contains but a single constant; and that, this constant having been determined by means of the measures of Biot in one class of phenomena, the author has been enabled to calculate the results in another and totally distinct class, observed by Mr. Airy,

and has found the results of calculation and those of observation to agree. It still remains, however, to deduce from mechanical principles the differential equations assumed by Professor Mac Cullagh; when this shall have been accomplished, the solution of this perplexing problem will be complete.

The phenomena of depolarization and of colour, impressed by double-refracting substances upon the transmitted light, are, we have seen, the necessary results of the interference of the two pencils into which the light is divided within them. These properties, therefore, become distinctive characters of the double-refracting structure, and thus enable us to discover the existence, and to trace the laws of that structure, even in substances in which the separation of the two pencils is too minute to be directly observed. By such means the important discovery has been made, that a double-refracting structure may be communicated to bodies which do not possess it naturally, by *mechanical compression or dilatation*. Thus Sir David Brewster observed, that when pressure was applied to the opposite faces of a parallelopiped of glass, it developed a tint in polarized light, like a plate of double-refracting crystal; and the tint descended in the scale as the pressure was augmented. Single-refracting crystals, such as muriate of soda and fluor spar, acquired the property of double refraction by the same means.

This is in perfect accordance with the wave-theory. Owing to the connexion of the vibrating medium with the solid in which it is contained, its elasticity is rendered unequal in different directions by the effect of compression, the maximum and minimum of elasticity corresponding to the directions of greatest and least pressure. Accordingly, the vibrations of the ray, on entering the substance, are resolved into two in these directions, and these are propagated with unequal velocities. One of the rays, therefore, will be retarded more

than the other, and they will be in different phases of vibration at emergence; the resolved parts of these vibrations in the plane of reflexion of the analyzing plate will accordingly interfere, and the tint developed will be determined by the interval of retardation.

These results of theory were experimentally confirmed by Fresnel; and he found that the velocity with which a ray traversed the glass was greater or less, according as its plane of polarization coincided with, or was perpendicular to, the line in which the pressure was exerted. The double refraction of the ray is a necessary consequence of this difference of velocities;—but this was also established by Fresnel by *direct* experiment. A series of glass prisms were placed together, with their refracting angles alternately in opposite directions, and the ends of the alternate prisms powerfully pressed by screws: a ray transmitted through the combination was found to be divided into *two oppositely polarized*. The compressed prisms, in this arrangement, had acquired a double-refracting structure, the axis of pressure being also the axis of double refraction; and their refracting angles being all turned in the same direction, the divergence of the two rays was increased in proportion to their number, and thus rendered sensible. The intermediate prisms served to correct the deviation, and to render the combination achromatic.

The opposite effects of compression and dilatation may be very well seen, and studied, in a thick plate of glass *bent* by an external force. The entire mass of the plate is thus thrown into an altered state of density, the parts towards the *convex* side of the plate being *dilated*, and those towards the *concave* side *compressed*; while, about the middle of the thickness, there is a surface in which the particles are in their natural state. Accordingly, when this body is interposed between the polarizing and analyzing plates, so as to form an angle of 45° with the plane of primitive polarization, two sets of

coloured bands are seen, separated by a neutral line; and these vanish altogether when the compressing force is withdrawn. The parts towards the *convex*, or dilated side of the neutral line, are found to have acquired a *positive* double-refracting structure, and those on the *concave*, or compressed side, a *negative* one.

In these cases of induced double refraction, the phenomena are related to the form of the entire mass; and the axes of elasticity are single lines within the substance, fixed in *position*, as well as *direction*. In this respect the phenomena are essentially different from those produced by *regular crystals*, in which the elasticity and double refraction depend solely on the *direction*, and are the same in all parts of the substance.

The effects of unequal compression and dilatation may be much more regularly produced by the application of heat. These effects may be studied by applying a bar of hot iron to the edge of a rectangular plate of glass, and placing it in the polarizing apparatus, so that the heated edge may form an angle of 45° with the plane of primitive polarization. At the end of some time, the whole surface of the plate will be observed to be covered with coloured bands, the parts near the opposite edges having acquired a positive double-refracting structure, and those near the centre a negative one. The effects are reversed when a plate of glass, uniformly heated, is rapidly cooled at one of its edges; and all the appearances vanish when the glass acquires the same temperature throughout.

If we transmit heat from the surface to the axis of a glass cylinder, by immersing it in heated oil, it will display a system of rings similar to those of a negative crystal with one axis, the axis of the cylinder being also the axis of double refraction. When the heat reaches the axis, the double refraction begins to weaken, and the colours disappear altogether when the glass is uniformly heated. Again if the cylinder,

when in this state, be made to cool rapidly, by surrounding it with a good conductor of heat, it will transiently assume the opposite character of a positive double-refracting crystal ; and when it becomes of a uniform temperature throughout, all traces of double refraction again disappear. If we employ an *elliptic* cylinder, instead of a circular one, in the experiment just described, it will exhibit the coloured curves formed by a *biaxal* crystal : and the phenomena may be endlessly varied by varying the form of the glass to which the heat is applied.

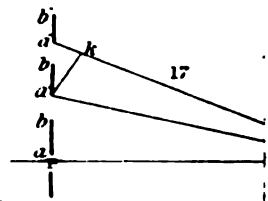
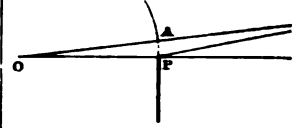
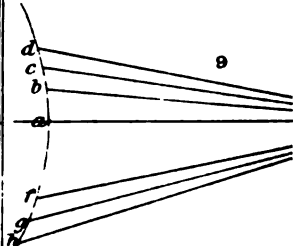
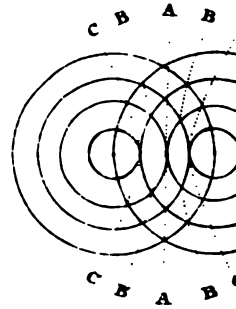
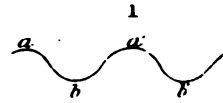
If now, by any means, the glass be arrested in one of these transient states, it will acquire a *permanent* double-refracting structure. This has been accomplished by raising it to a red heat, and then cooling it rapidly at the edges ; for, as the outer parts, which are thus more condensed, assume a fixed form in cooling, the interior parts must accommodate themselves to that form, and therefore retain a state of unequal density. The law of density, and therefore the double-refracting structure, will depend on the external form ; and it is accordingly found that the coloured bands and patches, which such bodies display in polarized light, assume a regular arrangement varying with the shape of the mass.

As the double-refracting structure is communicated to bodies which do not possess it naturally, by mechanical compression, or unequal temperature,—so, by the same means, that structure may be *altered* in the bodies in which it already resides. Thus Sir David Brewster and M. Biot found that the double refraction of *regular crystals* may be altered, and the tints they display made to rise or descend in the scale, by *simple pressure*. But the changes induced by *heat* are more remarkable. Professor Mitscherlich discovered the important fact, that, in general, heat dilates crystals *differently in different directions*, and so alters their form ; and their double-refracting properties are found to undergo a corresponding change. Thus, Iceland spar is dilated by heat in the direction of its axis ; while it actually *contracts*, by a small

amount, in directions perpendicular to it. The angles of the primitive form thus vary, the rhomboid becoming less obtuse, and approaching the form of the *cube*,—in crystals of which form, we have already seen, there is no double refraction. Professor Mitscherlich accordingly conjectured, that the double-refracting energy of the crystal must, in these circumstances, be diminished; and the conjecture was verified by experiment. In fact, the extraordinary index in Iceland spar increases considerably with the temperature, while the ordinary index undergoes little or no change.

We have seen that the inclination of the optic axes, in biaxial crystals, is a simple function of the three principal elasticities of the vibrating medium; the plane of the axes being that of the greatest and least elasticities. If then, these elasticities be altered by heat in *different proportions*, the inclination of the axes will likewise vary; and it may even happen that the plane of the axes will shift to a position at right angles to that which it formerly occupied. All these variations have been actually observed. Professor Mitscherlich found that, in sulphate of lime, the angle between the axes (which is about 60° at the ordinary temperature) diminishes on the application of heat; that, as the temperature increases, these axes approach until they unite; and that, on a still further augmentation of heat, they again separate, and open out in a perpendicular plane. Heat is found to dilate this crystal more in one direction than in another perpendicular to it. Analogous phenomena have been observed by Sir David Brewster in glauuberite.

Lectures on Waves





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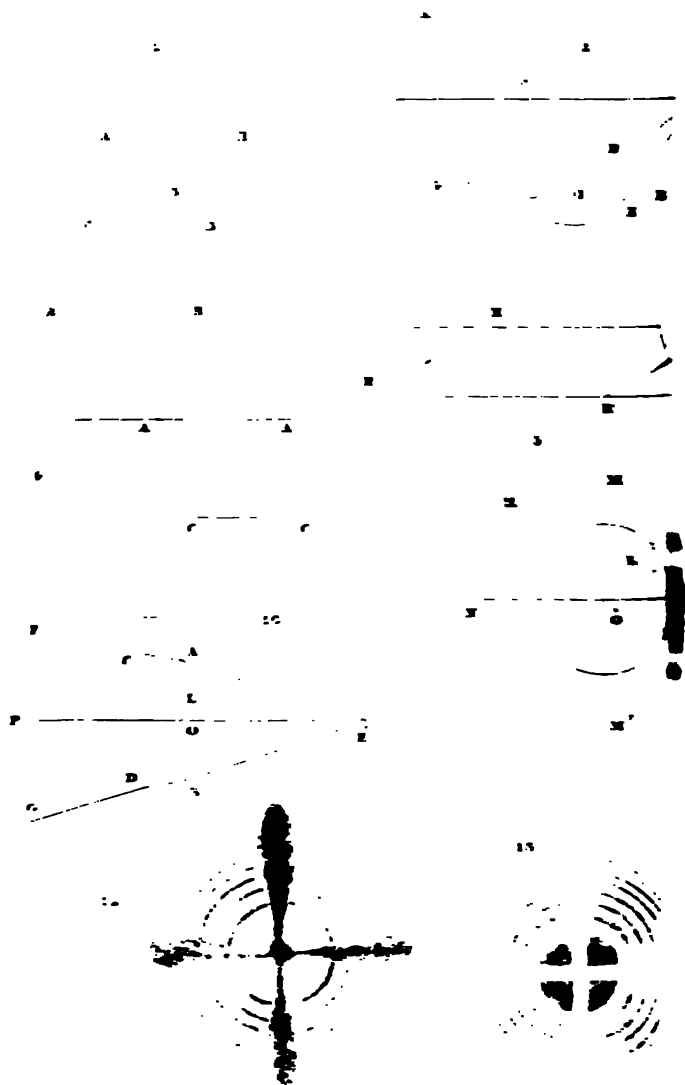
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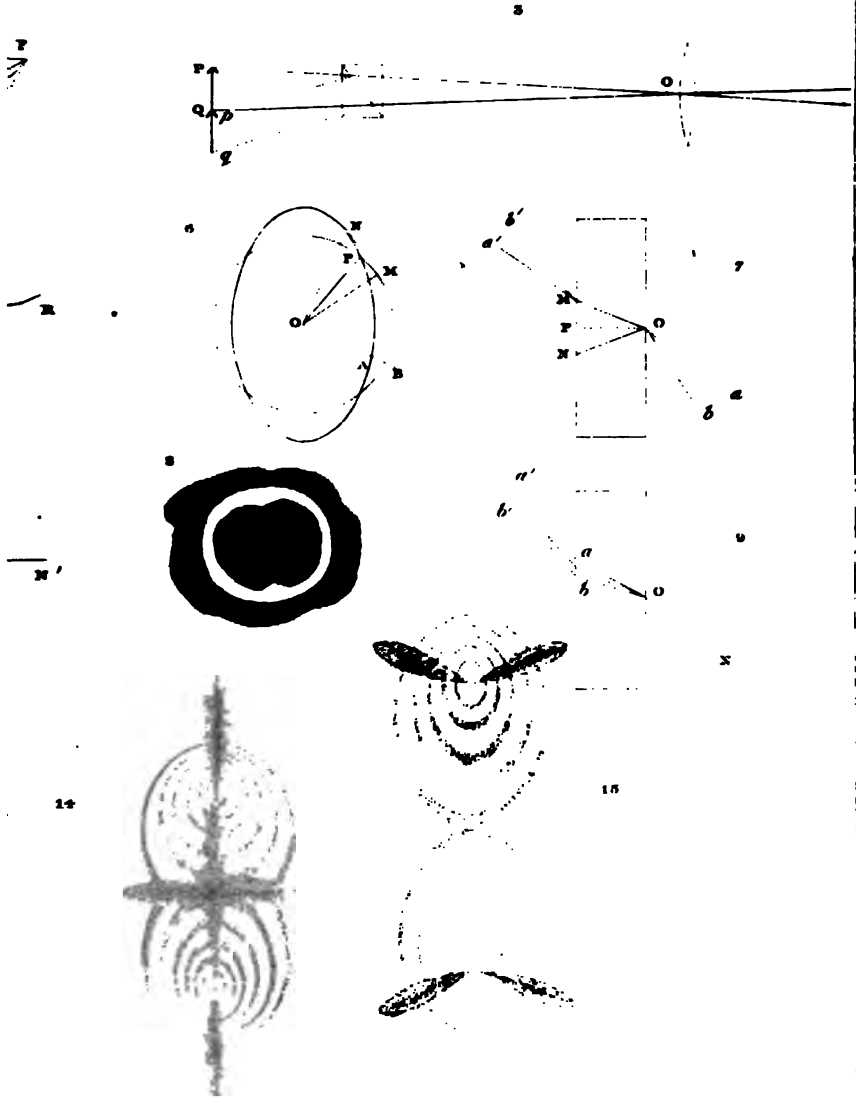
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THEORY OF THE EARTH





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